PROGRAM METHOD AND USAGE DOCUMENT

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BUCLASP

A Computer Program for Uniaxial Compressive Buckling Loads of Orthotropic Laminated Stiffened Plates

Prepared for the

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ABSTRACT

This Program Method and Usage document summarizes the analytic method and describes the usage of a CDC 6600 FORTRAN IV digital computer program which uses minimum energy principles to solve for compressive buckling loads and displacement pattern of stiffened plates built up of orthotropic laminated flat plate elements as well as beam elements (e.g. integral panels, corrugated plates). The program takes advantage of the repetitive nature of the problem, and only one of the repetitive parts and the end parts of the stiffened plate need to be described in the input together with the number of repetitions. The program can handle a maximum of 25 plate elements with up to 25 laminas and 10 beam elements with up to 35 laminas. The plate cannot be of more than 10 different types in the sense that their stiffnesses are the same. When the geometry and material constants are given the program will calculate the buckling load and optionally also the displacement pattern for the chosen boundary conditions. The unloaded sides can be free, simply supported, or clamped or be supported by a beam element. The loaded edges are simply supported. A correlation between results from this program and the literature is shown. A Program Description Document is also available for this program. The program is developed for NASA, Langley Research Center, under Contract No. NAS1-8858.

Additional documents under this contract are:

(1) Program Method and Usage Document; (2) Program Description Document: BUCLAP - "A Computer Program for Uniaxial Compressive Buckling Loads of Orthotropic Laminated Plates."

(3) Program Method and Usage Document; (4) Program Description Document: BUCLAS - "A Computer Program for Uniaxial Compressive Buckling Loads of Orthotropic Laminated Structural Sections."

(5) Analysis Report - "Buckling Analysis for Axially Compressed Flat Plates, Structural Sections and Stiffened Plates Reinforced with Laminated Composites.

KEY WORDS

BUCKLING	LAMINATES
STIFFENED PLATES	ORTHOTROPIC
COMPOSITES	PLATE ELEMENTS
STRUCTURAL SECTIONS	SANDWICH PLATES
BUCKLING DISPLACEMENTS	EIGENVECTOR

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1.0 SUMMARY

The BUCLASP program has the capability to solve for the critical compressive buckling load on stiffened plates, e.g. corrugated plates, integral panels, built up of orthotropic laminated, flat plate elements as well as beam elements. Optionally the program will also establish the buckling displacement pattern for the critical load and mode. The method used here is the classical approach of minimum energy consideration. Linearized theory is used. The general othotropic, laminated plate theory that was derived earlier for the plate program BUCLAP is applied to each plate element of the cross-section of the stiffened plate in a similar way as for the program for buckling of structural sections BUCLAS. Full compatibility is satisfied at the junctions of the plate elements. Thus the boundary conditions for each plate element will contain forces and displacements of the adjacent plate elements.

1.1 Problem Description

This program was originated in connection with NASA Contract No. NAS1-8858. The purpose of the computer program is to implement the analytical work under the same contract.

The objective of this effort is to develop a program which computes the axial compressive buckling loads for various composite reinforced stiffened panels. The cross-section is built up from the required number of flat laminated plate elements and also beam elements of various shapes. The boundary conditions at the loaded edges of each element are simply supported, while the boundary conditions at the unloaded sides of the cross-section can be free, simply supported or clamped. These unloaded sides can also be supported by beam elements; e.g. lips or beads.

This program is the third in a series of programs for buckling of composites, where the first program concerns the buckling loads at plates with various boundary conditions, and the second involves the solution of buckling loads for structural sections.

1.2 Results

The numerical results obtained from the programmed method is correlated with the available literature for isotropic stiffened panels, for the various boundary conditions. The effect of assuming certain plate elements as beam elements is studied.

- For orthotropic laminated stiffened plates the results available in the literature are limited, but the data available is checked. Sample cases of isotropic and orthotropic plates are simulated to run on this program and the results are compared to the results obtained from the plate program BUCLAP and the sections program BUCLAS (both developed under this contract) for the same data.

As can be seen from Section 5.0 the correlation is good.

The functional aspects are tested as shown in Section 5.2.

1.3 Conclusions

The quality of the verification of results (Section 5.0) demonstrates that the theoretical method is adequate.

1.4 Recommendations

The numerical difficulties, inherent in the type of problem solved here, have established the search strategy for determining the critical load, which involves finding the zero crossing of the buckling determinant. The progress of this search depends upon the magnitude of the starting load and a load interval. One property of high order polynomials (i.e. large determinants) is that its roots will be very close together, thus in certain cases two, or more, zero crossings of the determinant may occur for quite close buckling loads. Also in certain cases the critical load is very close to a load which gives double roots when the equilibrium equations are solved. Under conditions like these, care must be exercised in choosing the starting load and the iteration step size. If convergence difficulties are encountered, adjustment (decrease) of the load increment input data normally increases the likelihood of successful solution achievement.

The program has been coded with care so as to minimize the probability for any of these problems to occur.

2.0 THEORY

In this section, only those parts and details necessary for the understanding of the computer program, are given. Full details of the analysis are given in: Viswanathan, A. V.; Soong, T. C.; and Miller, Jr., R. E.: "Buckling Analysis for Axially Compressed Flat Plates, Structural Sections and Stiffened Plates Reinforced with Laminated Composites," prepared for NASA Langley Research Center, by The Boeing Company, November, 1970.

2.1 Notation

a	Axial length of the structure.
$A_{11}, A_{12}, A_{22}, A_{66}$	Extensional stiffnesses
A_b	Area of beam element
b	Width of beam element
$B_{11}, B_{12}, B_{22}, B_{66}$	Bending-stretching coupling stiffnesses
$D_{11}, D_{12}, D_{22}, D_{66}$	Bending stiffnesses
$E_{11}, E_{12}, G_{12}, G_{23}$	Modulii of elasticity
h_k	Coordinate for the surface of k^{th} lamina (Figure 2.3)
h'_k	Distance from neutral plane to the surface of k^{th} lamina (Figure 2.3)
I_{yy}, I_{zz}	Moments of inertia of beam element
J	St. Venant torsion constant
$K_8, K_6, \ldots, K_0$	Coefficients of characteristic equation (Equation (20))
ℓ	Number of laminas
L_{1i}, L_{2i}, L_{3i}	Displacement ratios (Equations (21), (22), and (24))
m	Axial half-wave number
$(m_{22})_i, (m_{22})_{wi}$	Moment factors (Equations (33) and (41))
M_{11}, M_{12}, M_{22}	Moments due to buckling displacements
$(n_{12})_i, (n_{12})_{ui}$	In-plane shear factors (Equations (34) and (42))
$(n_{22})_i, (n_{22})_{ui}$	In-plane force factors (Equations (35) and (43))
N_{11}, N_{22}, N_{12}	In-plane forces due to buckling displacements
$\bar{N}_{11}$	External uniaxial compressive load on flat plate elements (lbs/in.)
p_i, p_{ui}, p_{wi}	Roots of characteristic equation

$\overline{P}_b$	External axial compressive load on beam elements (lbs)
P_u	Axial load in beam elements induced by buckling displacements.
q_i, q_{wi}	Transverse shear factors (Equations (32) and (40))
q_y, q_z	Transverse shear intensity in beam elements
Q	Transverse shear in flat plate elements
Q_{11}^k, Q_{22}^k, etc.	Elements of lamina stiffness matrix
$\{R_1\}, \{R_2\}, \{R_1^*\}$	Displacement coefficient matrices
$R_{11}, R_{12}, \ldots, R_{33}$	Elements of flat plate element equilibrium matrix
S_{11}^k	First element of the lamina compliance matrix, $[Q_{ij}^k]^{-1}$
t_k	Thickness of k^{th} lamina
$[T_d], [T_f]$	Transformation matrices
T_x	Torque on beam element
u, v, w	Neutral plane displacements of the plate and the center of the beam element
x, y, z	Local coordinates of elements
$[X_1], [X_1^*]$, $[X_3^{\pm}], [X_3^{*\pm}]$, etc.	Matrices for displacements and forces of flat plate and beam elements for inter-element matching.
y_o, z_o	Off-sets
z_n	Distance to neutral plane of flat plate elements (Figure 2.3)
[ ]	Matrices
{ }	Column matrix
$(\ )_{,x}$, etc.	$\frac{\partial(\ )}{\partial x}$, etc.

β, δ	Wave mode parameters (Equations (16) and (80))
Γ	Warping constant of beam element
ϵ, γ	In-plane unit strains
θ	Rotation about longitudinal axis
ξ_1 to ξ_4	Beam element force factor (Equation (83))
$\sigma_x, \sigma_y, \sigma_{xy}$	In-plane stress components
$\bar{\sigma} I_p$	Beam element property
ψ	Angle between global Y and local y axes
$\omega_i, \phi_i, \eta_i, \rho_i$ $\omega_{wi}, \phi_{wi}, \eta_{ui}, \rho_{ui}$	Displacement factors for flat plate elements. Equations (28) to (31) and Equations (36) to (39).

Superscripts:

+	Side y = + b/2 of flat plate element
-	Side y = - b/2 of flat plate element

Subscripts:

BG	Beam element quantities with respect to global axes
BS	Beam element quantities with respect to local off-set axes.
(1), (2), etc.	Element numbers
i	Numbering of characteristic equation roots of flat plate elements.
k	Lamina numbering
PG	Flat plate element quantities with respect to global axes.
PS	Flat plate element quantities with respect to local off-set axes.

2.2 Method

The type of structure, considered is of uniform cross-section and is assembled from orthotropic laminated flat plate and laminated beam elements. In a macro-mechanic sense, the material in each lamina is homogeneous and orthotropic with respect to the axes of the structure. This restriction on orthotropicity can be relaxed, without causing serious error under certain conditions, as discussed in Section 2.3.2.

The loading is uniaxial compression. Typical examples of such structures are stiffened plates, truss core and corrugated core sandwich plates, etc. The method of numerical solution used precludes any "closed" structure, where the first and the last elements are interconnected as in a polygonal box. However, a closed part can be part of the whole, as in hat stiffened plates, truss core sandwich plates, etc. The numerical method also takes advantage of any repetitive nature of part of the structure (e.g. repetitive stiffeners in stiffened plates).

The intersecting angle between the elements can be arbitrary. The loaded edges of each element are simply supported. Any unloaded edge of the structure, when not supported by a beam element can be free, simply supported or clamped.

In the present buckling analysis linear theory is used. The prebuckling deformations and possible initial imperfections are ignored. The buckling load is defined as the smallest load at which a part of the structure (local instability) or the whole of the structure (general instability) starts to develop out-of-plane displacements (w), resulting in a state of unstable equilibrium consistent with the given boundary conditions.

A set of buckling displacement functions, automatically satisfying the simply supported boundary conditions along the loaded edges and having the same axial mode (wave-length) in all elements, are assumed for each element making up the structure.

For flat plate elements, the buckling displacements assumed are u, v, and w. Substitution of the displacement functions into the equilibrium equations for a laminated flat plate leads to a characteristic equation for each element, which in general is a polynomial of 8th degree. Corresponding to each level of uniform axial strain in all elements, there is a set of roots from this characteristic equation, for each flat plate element. Using these roots, buckling displacements and the corresponding forces along the unloaded edges of each flat plate element are evaluated.

For beam elements, the buckling displacements assumed in the translations u, v, w, and the rotation θ about the longitudinal axis. Applying the theories of bending and torsion of beams (including axial compression effects) the forces along the beam due to buckling displacements are determined.

In a structure made up of these elements, for continuity, the corresponding displacements of adjacent elements should be equal. Similarly, since there are no external loading at a junction of elements, for equilibrium, the corresponding forces from adjacent elements should be equal and opposite. These enforced continuity and equilibrium requirements form the boundary conditions along the junctions of elements. Along any unloaded edge, when not supported by a beam element, the boundary conditions can correspond to a free, simply supported or clamped edge.

These enforced boundary conditions result in a set of homogeneous simultaneous equations. The buckling load is obtained from these equations by determining the minimum value of the applied load for which the determinant of the coefficient matrix becomes zero. Buckling loads corresponding to various modes in the axial direction are evaluated and the minimum determined. The eigenvector giving the buckled shape corresponding to the minimum buckling load is obtained using inverse iteration technique.

It is important to note that the buckling loads and the corresponding eigen-modes are determined from a general instability analysis, in that no restrictions are placed on the buckling deformation of the cross-section (except that the angles between elements remain unchanged). The eigen-modes are indicative of overall or local nature of instability. In contrast the classical buckling analysis assumes restricted deformation of the cross-section as in flexural (Euler) mode, torsional mode, local mode, etc. Such simplifying restrictions can sometimes result in missing the lowest buckling load.

Attention is also drawn to the fact that the loaded edge of each element making up the structure is assumed to be simply supported. Thus each flat plate element has a line condition of simple support and each beam element a point condition of simple support at the loaded edges. Thus for structures of complex cross-sectional shape, the overall end conditions in the present analysis will be different compared to the conventional Euler instability theory, where the structure as a whole is idealized to a line member and simply supported, resulting in a point condition of simple support at the loaded ends. The effect of this will be small when the axial half-wave length of buckling is small compared to the length of the structure.

2.3 Idealization of the Structure under Uniaxial Compression into Elements

This section describes how the structure is idealized as an assemblage of flat plate elements and beam elements. As stated in Section 2.2, the structure is of uniform cross-section. Figure 2.1 shows the cross-section of a typical arbitrary structure. Y, Z axes are the global axes, the global X-axis being parallel to the longitudinal axis of the structure. The dashed line represents the outer contour of the structure. The solid line in the interior of the structure is drawn through the mid-plane of each segment.

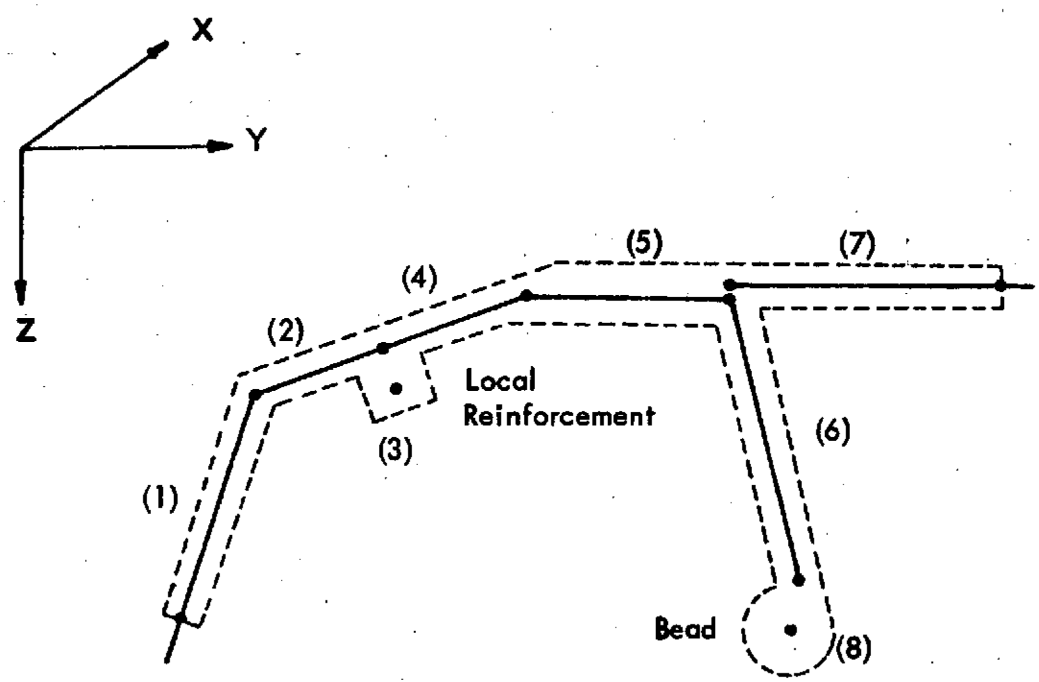


FIGURE 2.1. Idealization of Structure into Elements

It is easy to see that the flat sides of the structure are idealized as flat plate elements and the bead and the local reinforcement as beam elements. The numbers in parenthesis are the element numbers.

The intersection of the mid-plane lines for adjacent segments is used to identify the boundaries of individual flat plate elements. These intersections are indicated by the dots in Figure 2.1, where the dots are also used to identify the geometric center of the beam elements.

When three or more flat plate elements of differing thicknesses are involved at a junction (e.g. junction of elements (5), (6), and (7) in Figure 2.1) it is not possible to have a common intersection of all the mid-planes. In such cases the intersection of two or them are chosen while the other flat plate elements are assumed to have fictitious rigid off-sets to the chosen intersection. These off-sets are further discussed in Section 2.4.4.

The geometric centers (beam elements) and the mid-planes (flat plate elements) are chosen since the element junctions can be easily fixed from the geometry alone. No other special significance is attached to this choice.

The structure of Figure 2.1 can be considered as an assemblage of:

(a) beam elements (3) and (8),
(b) flat plate elements (1), (2), (4), (5), (6), and (7).

Any other structure of uniform cross-section can be idealized in a similar manner. Figure 2.2 shows a hat stiffened plate, with the element numbers shown in parenthesis. Attention is drawn to element number (2) which is a typical flat plate element, formed in this case by part of the skin and the attached flange. This idealization which is used in the present program, is correct when the stiffeners are bonded to the skin.

When stiffeners are rivetted to the skin this idealization over-estimates the bending stiffness of such elements.

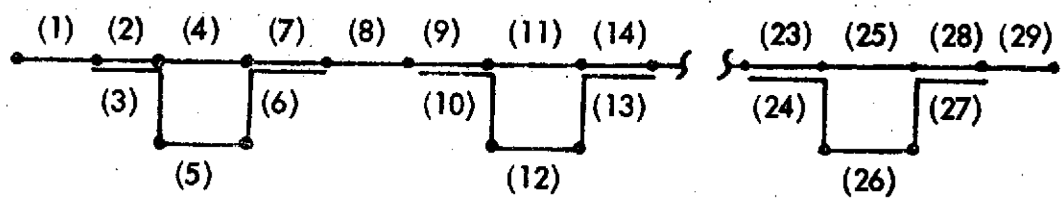


FIGURE 2.2. Idealization of a Hat Stiffened Plate into Elements

It is possible to idealize each complete stiffener in a stiffened plate as a beam element, using the appropriate beam properties, discussed in Section 2.5.1 and 2.5.2. Since the beam elements have a point simple support condition at the loaded ends and the flat plate elements have a line condition of simple support along the loaded edges, the idealization of each complete stiffener as a beam element will result in different overall end conditions for the stiffened plate. This may result in differing buckling loads. Further, when the stiffeners are of thin plate construction (e.g. hats, zees, etc.), the instability analysis based on the beam idealization of each complete stiffener, naturally will not cover local instability of any part of the stiffener.

In the buckling analysis, the element displacements and forces (due to buckling displacements) satisfy conditions of inter-element continuity and equilibrium, together with any other specified boundary conditions along the unloaded edges not supported by a beam element.

2.4 Flat Plate Element

Any flat part or side of the structure under uniaxial compression, which behaves as a plate is defined here as a flat plate element. The flat plate element is of constant thickness and is, in general, laminated. As stated earlier each lamina is orthotropic with reference to the axes of the structure. This restriction on orthotropicity can be relaxed without causing serious error under certain conditions, as discussed in Section 2.4.2. The basic equations of the flat plate element are given in the following subsections.

2.4.1 Material and Geometry Constants for a Flat Plate Element

For a lamina of orthotropic material, the matrix Q relates stress and strain in the following manner:

$$\{\sigma\} = [Q]\{\epsilon\} \tag{1}$$

or

$$\begin{Bmatrix} \sigma_x^k \\ \sigma_y^k \\ \sigma_{xy}^k \end{Bmatrix} = \begin{bmatrix} Q_{11}^k & Q_{12}^k & 0 \\ Q_{12}^k & Q_{22}^k & 0 \\ 0 & 0 & Q_{66}^k \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{2}$$

where k denotes lamina number.

The Q-matrix for a lamina depends on the material properties of the lamina,

$$
\begin{aligned}
Q_{11} &= \frac{E_{11}}{(1 - \nu_{21}\nu_{12})} \\
Q_{22} &= \frac{E_{22}}{(1 - \nu_{21}\nu_{12})} \\
Q_{12} &= \frac{\nu_{21}E_{11}}{(1 - \nu_{21}\nu_{12})} = \frac{\nu_{12}E_{22}}{(1 - \nu_{21}\nu_{12})} \\
Q_{66} &= G_{12}
\end{aligned}
\tag{3}
$$

Note that x, y, and z axes are assumed to be identical with directions 1, 2, and 3, respectively.

The quantity z_n locates a neutral plane relative to the reference plane chosen at either one of the outer surfaces of the flat plate element. This neutral plane is fixed by locating the resultant of the uniaxial force in the layers for a uniform strain across the thickness. Whenever the matrix [B] = 0 (coupling stiffness, see page 2.14), with respect to this neutral plane, there is no coupling between bending and stretching (for instance, an isotropic or a symmetrically laminated flat plate element).

The expression for z_n is:

$$
z_n = \frac{\sum_{k=1}^{\ell} \frac{1}{2} \frac{1}{S_{11}^k} t_k(h_{k+1} + h_k)}{\sum_{k=1}^{\ell} t_k/S_{11}^k}
\tag{4}
$$

where S_{11}^k is the first element of $[Q]^{-1}$ for layer number k.

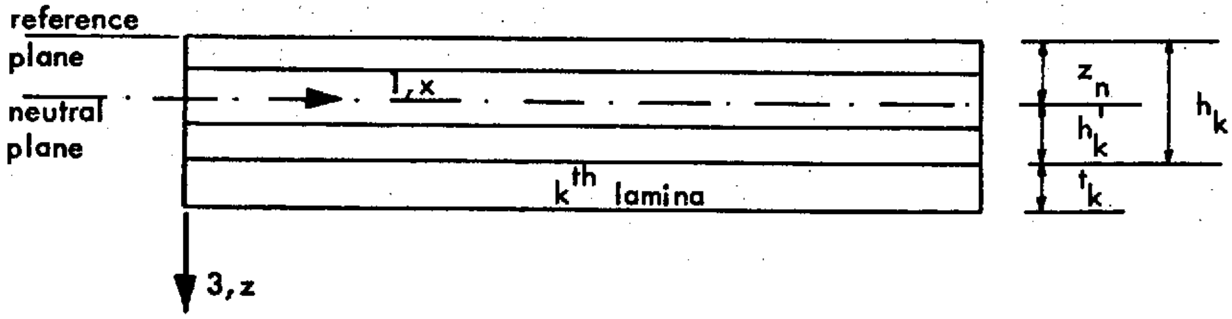


Figure 2.3. Flat Plate Element: Laminate Geometry

2.4.2 Overall Stiffnesses of a Flat Plate Element

The Q-matrix mentioned above (Equations 2 and 3) represents the stiffness matrix for each lamina. In the actual calculations, the stiffnesses of the flat plate element as a whole are needed.

In the following, the extensional stiffness, coupling stiffness and bending stiffness for the flat plate element are denoted by the A-matrix, B-matrix, and D-matrix, respectively.

Using linear theory, the strain at any point across the thickness in terms of the displacements, is written as:

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} u_{,x} \\ v_{,y} \\ u_{,y} + v_{,x} \end{bmatrix} - z \begin{Bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{Bmatrix} \qquad (5)$$

where z = distance from point to neutral plane

u, v, and w are the displacements of the neutral plane.

Thus the stresses $\{\sigma\}$ due to any displacements u, v, and w at any point, can be calculated using Equations (5) and (2). The stresses and their moments can be integrated across the thickness to establish the force and moment resultants on the differential element.

$$
\begin{aligned}
N_{11} &= \int_t \sigma_x dz & M_{11} &= \int_t \sigma_x \cdot z \cdot dz \\
N_{22} &= \int_t \sigma_y dz & M_{22} &= \int_t \sigma_y \cdot z \cdot dz \\
N_{12} &= \int_t \sigma_{xy} dz & M_{12} &= \int_t \sigma_{xy} \cdot z \cdot dz
\end{aligned}
\tag{6}
$$

The integration yields:

$$
\begin{Bmatrix} N_{11} \\ N_{22} \\ N_{12} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} u_{,x} \\ v_{,y} \\ u_{,y} + v_{,x} \end{Bmatrix} - \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} \begin{Bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{Bmatrix} \tag{7}
$$

$$
\begin{Bmatrix} M_{11} \\ M_{22} \\ M_{12} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} \begin{Bmatrix} u_{,x} \\ v_{,y} \\ u_{,y} + v_{,x} \end{Bmatrix} - \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{Bmatrix} \tag{8}
$$

where the A, B, and D coefficient matrices define the extensional, coupling and bending stiffnesses, respectively, of the laminated orthotropic flat plate element.

The elements of the A, B, and D matrices are given by:

$$A_{ij} = \sum_{k=1}^{\ell} (Q_{ij})_k \cdot t_k \tag{9}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{\ell} (Q_{ij})_k \, (h'_{k+1} + h'_k) \cdot t_k \tag{10}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{\ell} (Q_{ij})_k \, (h'^2_{k+1} + h'_{k+1} \cdot h'_k + h'^2_k) \cdot t_k \tag{11}$$

In the above expressions, k is the lamina number, ℓ is the total number of laminas. h'_k is the distance from the neutral plane to the surface of the respective lamina ($h'_k = h_k - z_n$).

Equations (2), (7), and (8) are based on the assumption that the material orthotropic axes coincide with the lamina axes. Boron fiber reinforced composite laminas with fibers at 0° or 90° are typical examples. When the fibers are at any other angle, each lamina, though orthotropic with respect to the fiber direction, is anisotropic with respect to the lamina axes. Q_{ij} are then replaced by their transformed values $\overline{Q}_{ij}$ (see for example: Ashton, J. E.; Halpin, J. C.; Petit, P. E.: "Primer on Composite Materials," Technomic Publication, 1969, Equation (2-35)), resulting in the matrices [Q], [A], [B], and [D] in Equations (1), (7), and (8) being fully populated. However, for a mid-plane symmetric laminate composed of a large number of layers the "16" and "26" terms in these matrices are either zero or small and can be ignored.

2.4.3 Equations for a Flat Plate Element

The forces acting on a differential element of the flat plate element are shown in Figure 2.4.

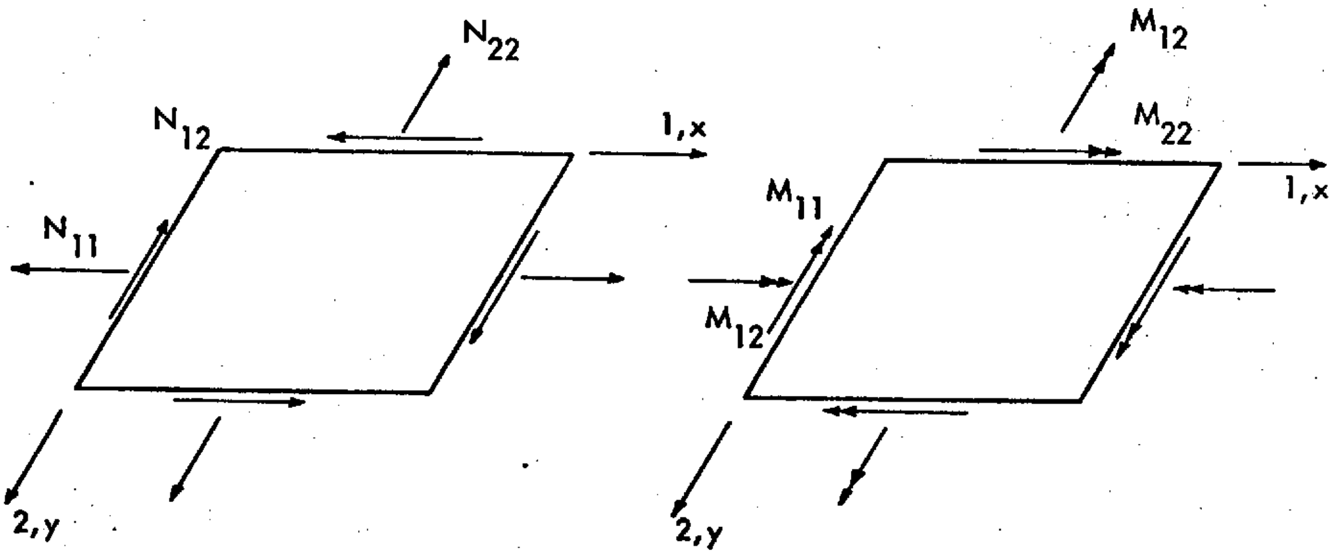


Figure 2.4. Forces on a Differential Element of the "Flat Plate Element"

Using variational principles, the three differential equations of equilibrium are obtained as:

$$\frac{\partial N_{11}}{\partial x} + \frac{\partial N_{12}}{\partial y} = 0 \tag{12}$$

$$\frac{\partial N_{22}}{\partial y} + \frac{\partial N_{12}}{\partial x} = 0 \tag{13}$$

$$\frac{\partial^2 M_{11}}{\partial^2 x} + \frac{\partial^2 M_{22}}{\partial^2 y} + 2\frac{\partial^2 M_{12}}{\partial x \cdot \partial y} = \overline{N}_{11} w_{,xx} \tag{14}$$

These equations can be expressed in terms of displacements using the relationship:

$$\begin{aligned} \{N\} &= [A]\{\epsilon^o\} + [B]\{\kappa\} \\ \{M\} &= [B]\{\epsilon^o\} + [D]\{\kappa\} \end{aligned} \tag{15}$$

where $\{\epsilon^o\}$ and $\{\kappa\}$ refer to strains and curvatures in the neutral reference plane.

The following, general buckling displacement functions are assumed for the flat plate element:

$$w = \sum_{i=1}^{8} W_i \sin \delta \cdot e^{\beta}$$

$$v = \sum_{i=1}^{8} V_i \sin \delta \cdot e^{\beta} \tag{16}$$

$$u = \sum_{i=1}^{8} U_i \cos \delta \cdot e^{\beta}$$

where

$$\delta = \frac{m\pi x}{a}$$

$$\beta = \frac{\pi P_i y}{a}$$

Figure 2.5 shows the geometry of the flat plate element. The x-y plane coincides with neutral plane of the plate. (Equation (4))

These functions automatically satisfy the simply supported boundary conditions along the edges x = 0 or x = a, where the external applied load acts.

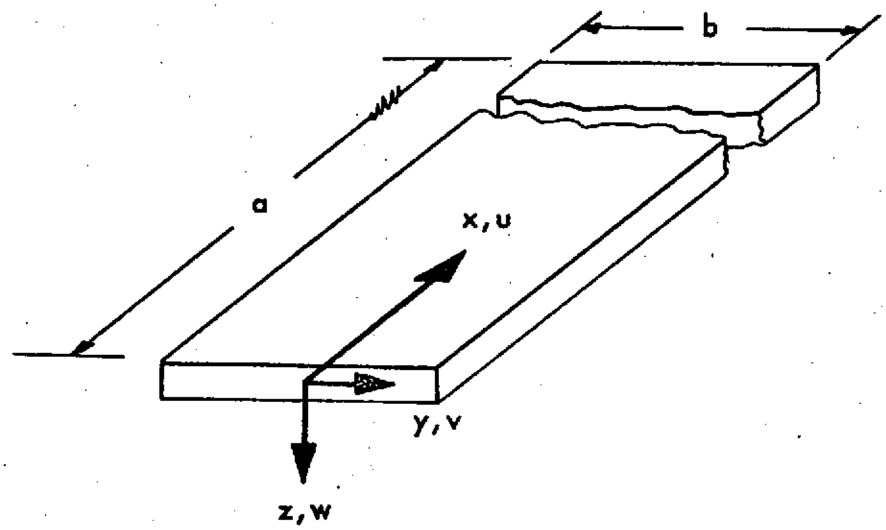


Figure 2.5. Flat Plate Element

On substituting the displacement functions assumed, the equilibrium equations reduce to:

$$\begin{bmatrix} R_{11} & R_{12} & \pi R_{13} \\ R_{21} & R_{22} & \pi R_{23} \\ R_{31} & R_{32} & \pi R_{33} \end{bmatrix} \begin{Bmatrix} U \\ V \\ W \end{Bmatrix} = 0 \tag{17}$$

A nontrivial solution is obtained from:

$$\left| D_T \right| = \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix} = 0 \tag{18}$$

where

$$R_{11} = -A_{11}\left(\frac{m}{a}\right)^2 + A_{66}\left(\frac{p_i}{a}\right)^2$$

$$R_{12} = (A_{12} + A_{66}) \cdot \left(\frac{m}{a}\right) \cdot \left(\frac{p_i}{a}\right)$$

$$R_{13} = B_{11}\left(\frac{m}{a}\right)^3 - (B_{12} + 2B_{66}) \cdot \left(\frac{m}{a}\right)\left(\frac{p_i}{a}\right)^2$$

$$R_{21} = -R_{12}$$

$$R_{22} = A_{22} \cdot \left(\frac{p_i}{a}\right)^2 - A_{66} \cdot \left(\frac{m}{a}\right)^2 \tag{19}$$

$$R_{23} = (B_{12} + 2 \cdot B_{66})\left(\frac{m}{a}\right)^2 \cdot \left(\frac{p_i}{a}\right) - B_{22}\left(\frac{p_i}{a}\right)^3$$

$$R_{31} = -R_{13}$$

$$R_{32} = R_{23}$$

$$R_{33} = -\overline{N}_{11} \cdot \frac{m^2}{\pi^2 a^2} + D_{11}\left(\frac{m}{a}\right)^4 - (2D_{12} + 4D_{66})\left(\frac{m}{a}\right)^2\left(\frac{p_i}{a}\right)^2 + D_{22} \cdot \left(\frac{p_i}{a}\right)^4$$

The determinant D_T of Equation (18) when expanded will yield an 8th order polynomial in p_i containing only even powers of p_i, i.e.

$$K_8 p_i^8 + K_6 p_i^6 + K_4 p_i^4 + K_2 p_i^2 + K_0 = 0 \tag{20}$$

Thus $\left|D_T\right| = 0$, when solved yields eight values of p_i, which are real or complex. When complex, they always appear as conjugate pairs. Using these p_i values in Equations (16), it is seen that the displacement functions satisfy the equilibrium equations of the flat plate element.

Also, through Equations (17), U_i and V_i in Equations (16) can be expressed in terms of W_i as:

$$U_i = \pi L_{2i} W_i \tag{21}$$

$$V_i = \pi L_{1i} W_i \tag{22}$$

where

$$L_{1i} = \frac{R_{23} \cdot R_{11} - R_{13} \cdot R_{21}}{R_{12} \cdot R_{21} - R_{22} \cdot R_{11}} \tag{21a}$$

and

$$L_{2i} = \frac{R_{13} \cdot R_{22} - R_{23} \cdot R_{12}}{R_{21} \cdot R_{12} - R_{22} \cdot R_{11}} \tag{22a}$$

It may be noted that when $B_{ij} = 0$ (i.e., no coupling between bending and stretching) the equilibrium Equations (12) and (13) contain only u and v terms and the Equation (14) contains only w terms. Thus, for this case u and v are independent of w. Instead of a single 8th order polynomial, Equation (20), one thus has:

(a) a separate fourth order polynomial in p_{ui} resulting from

$$\left|D_{TU}\right| = \begin{vmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{vmatrix} = 0 \tag{23}$$

The p_{ui} values are to be used for the displacements u and v of Equation (16), the summation extending from 1 to 4.

One can also express V_i in terms of U_i as

$$V_i = L_{3i} U_i \tag{24}$$

where

$$L_{3i} = -\frac{R_{11}}{R_{12}} = -\frac{R_{21}}{R_{22}} \tag{24a}$$

(b) another fourth order polynomial in p_{wi} resulting from

$$\left| D_{TW} \right| = R_{33} = 0 \tag{25}$$

The p_{wi} values are to be used for the displacement w of Equation (16), the summation extending from 1 to 4.

Using of these p_{ui} and p_{wi} values, thus satisfies the respective equilibrium equations of the flat plate element when $B_{ij} = 0$.

For a given level of axial load, $\bar{N}_{11}$ lb/in., in the flat plate element, the buckling displacements and the corresponding forces at any point in the neutral plane can now be readily calculated from Equations (16) and (15). When $B_{ij} \neq 0$, the p_i values from Equation (18) are used. When $B_{ij} = 0$, the p_{ui} values from Equation (23) are used in u and v displacements and the p_{wi} values from Equation (25) are used in the w displacement.

2.4.4 Forces and Displacements of Flat Plate Elements

In the buckling analysis the inter-element continuity and equilibrium are enforced along the junction line between the elements. These lines are in general off-set from the neutral plane and parallel to the longitudinal axis of the flat plate element. In this section the displacements and forces involved along the two sides of the flat plate element are considered in detail. They are initially derived with reference to the local coordinates in the neutral plane, from the equations given in Section 2.4.3. See Figure 2.6. They are then transferred to the off-set junction line. Finally they are transformed to the chosen global axes system.

The flat plate element displacements involved are:

$$w, \qquad w_{,y}, \qquad u, \qquad v \tag{26}$$

The plate element forces involved are:

$$Q = \left(\frac{\partial M_{22}}{\partial y} + 2\frac{\partial M_{12}}{\partial x}\right), \qquad M_{22}, \qquad N_{12}, \qquad N_{22} \tag{27}$$

Figure 2.6 shows these displacements and forces in the neutral plane along the side $y = +b/2$ of the flat plate element.

The forces in Equation (27), can be expressed in terms of the assumed buckling displacements through Equations (7), (8), and (16).

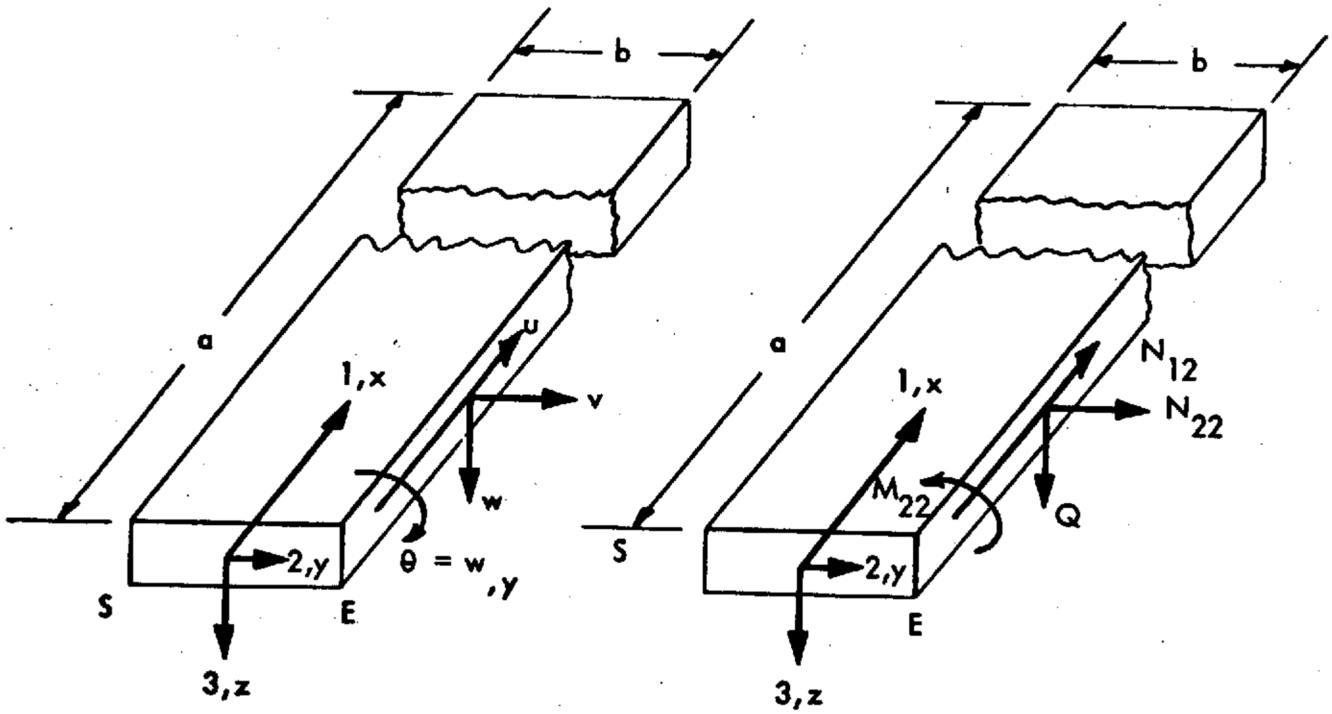


FIGURE 2.6. Flat Plate Element Displacements and Forces

When $B_{ij} \neq 0$ (i.e. there is coupling between bending and stretching) all the quantities can be expressed in terms of W_i (i = 1 to 8) through Equations (21) and (22). Also involved are the eight p_i values obtained from Equation (18). When $B_{ij} = 0$, (i.e. no coupling), as stated in Section 2.3.3, the u and v displacements are independent of w. In this case V_i can be expressed in terms of U_i. See Equation (24). Thus all the quantities can be expressed in terms of U_i and W_i (i = 1 to 4). Also involved are the four p_{ui} values obtained from Equation (23) and the four p_{wi} values obtained from Equation (25).

Finally, the displacements and forces involved in Equations (26) and (27) can be written as below:

<u>CASE 1.</u> $B_{ij} \neq 0$:

$$\delta = \frac{m\pi x}{a}$$

$$\beta = \frac{\pi p_i y}{a}$$

$$w = \sum_{i=1}^{8} W_i \sin\delta \cdot e^{\beta} = \sum_{i=1}^{8} \omega_i W_i \sin\delta \tag{28}$$

$$w_{,y} = \sum_{i=1}^{8} W_i \cdot \frac{\pi p_i}{a} \cdot \sin\delta \; e^{\beta} = \sum_{i=1}^{8} \phi_i W_i \sin\delta \tag{29}$$

$$u = \sum_{i=1}^{8} W_i \cdot \pi \cdot L_{2i} \cos\delta \; e^{\beta} = \sum_{i=1}^{8} \eta_i W_i \cos\delta \tag{30}$$

$$v = \sum_{i=1}^{8} W_i \cdot \pi \cdot L_{1i} \sin\delta \; e^{\beta} = \sum_{i=1}^{8} \rho_i W_i \sin\delta \tag{31}$$

$$Q = \sum_{i=1}^{8} [\{-B_{12} \cdot \frac{m}{a} \cdot \frac{p_i}{a} \cdot L_{2i} + B_{22}L_{1i}(\frac{p_i}{a})^2 + D_{12}(\frac{m}{a})^2 \cdot \frac{p_i}{a} \cdot$$

$$- D_{22}(\frac{p_i}{a})^3 - 2B_{66}(\frac{m}{a} \cdot \frac{p_i}{a} L_{2i} + (\frac{m}{a})^2 \cdot L_{1i})$$

$$+ 4D_{66}(\frac{m}{a})^2 \cdot \frac{p_i}{a}\} W_i \cdot \pi^3 \cdot e^{\beta} \cdot \sin \delta]$$

$$= \sum_{i=1}^{8} q_i W_i \sin \delta \tag{32}$$

$$M_{22} = \sum_{i=1}^{8} [\{-B_{12} \frac{m}{a} \cdot L_{2i} + B_{22} \frac{p_i}{a} L_{1i} + D_{12}(\frac{m}{a})^2 - D_{22}(\frac{p_i}{a})^2\} \cdot$$

$$W_i \cdot \pi^2 \cdot e^{\beta} \sin \delta]$$

$$= \sum_{i=1}^{8} (m_{22})_i W_i \sin \delta \tag{33}$$

$$N_{12} = \sum_{i=1}^{8} [\{A_{66}(\frac{p_i}{a} \cdot L_{2i} + \frac{m}{a} L_{1i}) - 2B_{66} \cdot \frac{p_i}{a} \cdot \frac{m}{a}\} \cdot W_i \cdot \pi^2 \cdot e^{\beta} \cdot \cos \delta]$$

$$= \sum_{i=1}^{8} (n_{12})_i W_i \cos \delta \tag{34}$$

$$N_{22} = \sum_{i=1}^{8} [\left\{ -A_{12} \frac{m}{a} \cdot L_{2i} + A_{22} \frac{p_i}{a} L_{1i} + B_{12}\left(\frac{m}{a}\right)^2 - B_{22}\left(\frac{p_i}{a}\right)^2 \right\} \cdot$$

$$W_i \cdot \pi^2 \cdot e^{\beta} \sin \delta]$$

$$= \sum_{i=1}^{8} (n_{22})_i W_i \sin \delta \tag{35}$$

The quantities defined by ω_i, ϕ_i, η_i, ρ_i, q_i, $(m_{22})_i$, $(n_{12})_i$, and $(n_{22})_i$ are self evident from the above equations ($B_{ij} \neq 0$).

CASE II. $B_{ij} = 0$:

$$\delta = \frac{m\pi x}{a}$$

$$\beta_1 = \frac{\pi P_{ui} y}{a}$$

$$\beta_2 = \frac{\pi P_{wi} y}{a}$$

$$w = \sum_{i=1}^{4} W_i \sin\delta \; e^{\beta_2} = \sum_{i=1}^{4} \omega_{wi} W_i \sin\delta \tag{36}$$

$$w_{,y} = \sum_{i=1}^{4} W_i \frac{\pi P_{wi}}{a} \sin\delta \; e^{\beta_2} = \sum_{i=1}^{4} \phi_{wi} W_i \sin\delta \tag{37}$$

$$u = \sum_{i=1}^{4} U_i \cos\delta \; e^{\beta_1} = \sum_{i=1}^{4} \eta_{ui} U_i \cos\delta \tag{38}$$

$$v = \sum_{i=1}^{4} L_{3_i} U_i \sin\delta \; e^{\beta_1} = \sum_{i=1}^{4} \rho_{ui} U_i \sin\delta \tag{39}$$

$$Q = \sum_{i=1}^{4} [\left\{ D_{12} (\tfrac{m}{a})^2 \cdot (\tfrac{P_{wi}}{a}) - D_{22} (\tfrac{P_{wi}}{a})^3 + 4D_{66} (\tfrac{m}{a})^2 \tfrac{P_{wi}}{a} \right\} W_i \cdot \pi^3 \cdot e^{\beta_2} \cdot \sin\delta]$$

$$= \sum_{i=1}^{4} q_{wi} W_i \sin\delta \tag{40}$$

$$M_{22} = \sum_{i=1}^{4} [\{D_{12}(\frac{m}{a})^2 - D_{22}(\frac{P_{wi}}{a})^2\} W_i \cdot \pi^2 \cdot e^{\beta_2} \sin \delta]$$

$$= \sum_{i=1}^{4} (m_{22})_{wi} W_i \sin \delta \tag{41}$$

$$N_{12} = \sum_{i=1}^{4} [\{A_{66}(\frac{P_{ui}}{a} + \frac{m}{a} \cdot L_{3_i})\} U_i \cdot \pi \cdot e^{\beta_1} \cdot \cos \delta]$$

$$= \sum_{i=1}^{4} (n_{12})_{ui} U_i \cos \delta \tag{42}$$

$$N_{22} = \sum_{i=1}^{4} [\{-A_{12} \frac{m}{a} + A_{22} \frac{P_{ui}}{a} L_{3_i}\} U_i \cdot \pi \cdot e^{\beta_1} \sin \delta]$$

$$= \sum_{i=1}^{4} (n_{22})_{ui} U_i \sin \delta \tag{43}$$

The quantities defined by ω_{wi}, ϕ_{wi}, η_{ui}, ρ_{ui}, q_{wi}, $(m_{22})_{wi}$, $(n_{12})_{ui}$, and $(n_{22})_{ui}$ are self-evident from the above equations ($B_{ij} = 0$).

Putting $y = \pm b/2$ in the above equations yield the displacements and forces along the two sides of the flat plate element, with reference to the local axes and in the neutral plane. The element width to be used in calculating these displacements and forces is evident from the idealization of the structure into elements, as discussed in Section 2.3.

The above displacements and forces are now transferred to the inter-element junction line, which in general is off-set from the neutral plane. This transformation is purely a geometrical, rigid-body transfer.

Let y_o, z_o be the off-sets to this line measured positive in the positive directions of y and z axis, from the sides ($y = +b/2$ or $y = -b/2$) of the flat plate element.

These off-sets as defined above are measured from the neutral plane, as shown in Figure 2.7. Since the neutral plane is not initially known, the program input off-set in the z direction is measured from the "top surface" of the flat plate element. The "top surface" is the outer surface of the flat plate element on the negative z axis side. The program then internally calculates the true off-set z_o.

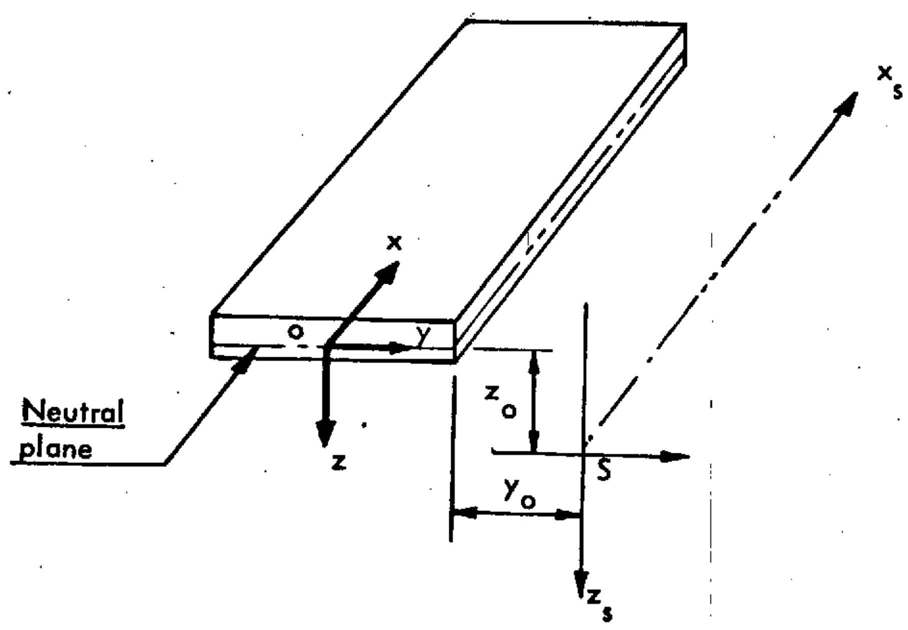


FIGURE 2.7. Off-Sets on Flat Plate Elements

A subscript 's' is used to designate the quantities with respect to the off-set axis. The displacements with respect to the off-set axis are then given by:

$$
\begin{aligned}
w_s &= w + y_o \cdot w_{,y} \\
\theta_s &= w_{s,y} = w_{,y} \\
v_s &= v - z_o w_{,y} \\
u_s &= u - z_o w_{,x} - y_o v_{,x}
\end{aligned}
\tag{44}
$$

The forces with respect to the same off-set axis are given by:

$$
\begin{aligned}
(M_{22})_s &= M_{22} + y_o Q - z_o N_{22} \\
(Q)_s &= Q - z_o \cdot N_{12,x} \\
(N_{12})_s &= N_{12} \\
(N_{22})_s &= N_{22} - y_o \cdot N_{12,x}
\end{aligned}
\tag{45}
$$

All the quantities in Equations (44) and (45) are defined by Equations (28) to (35) when $B_{ij} \neq 0$ and by Equations (36) to (43) when $B_{ij} = 0$, except $u_{,y}$, $w_{,x}$, and $N_{12,x}$. These are readily obtained after appropriate differentiation of u, w, and N_{12}. Thus when $B_{ij} \neq 0$:

$$
\begin{aligned}
v_{,x} &= \sum_{i=1}^{8} W_i \, \pi L_{1i} \left(\frac{m\pi}{a}\right) \cos\delta \; e^{\beta} = \sum_{i=1}^{8} W_i \left(\frac{m\pi}{a}\right) \rho_i \cos\delta \\
w_{,x} &= \sum_{i=1}^{8} W_i \cdot \left(\frac{m\pi}{a}\right) \cos\delta \cdot e^{\beta} = \sum_{i=1}^{8} W_i \left(\frac{m\pi}{a}\right) \cdot \omega_i \cos\delta
\end{aligned}
\tag{46}
$$

$$N_{12,x} = -\sum_{i=1}^{8} W_i \cdot [\left\{A_{66}(\frac{p_i}{a} \cdot L_{2i} + \frac{m}{a} L_{1i}) - 2B_{66} \frac{p_i}{a} \cdot \frac{m}{a}\right\} \cdot$$

$$\cdot \pi^2 \cdot (\frac{m\pi}{a}) \cdot \sin\delta \cdot e^{\beta}] \tag{46}$$

$$= -\sum_{i=1}^{8} W_i (\frac{m\pi}{a})(n_{12})_i \sin\delta$$

and when $B_{ij} = 0$:

$$v_{,x} = \sum_{i=1}^{4} L_{3i} U_i (\frac{m\pi}{a}) \cos\delta \; e^{\beta_1} = \sum_{i=1}^{4} U_i \; \rho_{ui} (\frac{m\pi}{a}) \cos\delta .$$

$$w_{,x} = \sum_{i=1}^{4} W_i (\frac{m\pi}{a}) \cos\delta \cdot e^{\beta_2} = \sum_{i=1}^{4} W_i (\frac{m\pi}{a}) \omega_{wi} \cos\delta$$

P (47)

$$N_{12,x} = -\sum_{i=1}^{4} U_i [\left\{A_{66}(\frac{pu_i}{a} + \frac{m}{a} \cdot L_{3i})\right\} \pi \cdot (\frac{m\pi}{a}) \sin\delta \cdot e^{\beta_1}]$$

$$= -\sum_{i=1}^{4} U_i (\frac{m\pi}{a})(n_{12})_{ui} \sin\delta$$

After making the above substitutions, Equation (44), giving the displacements of the flat plate element with respect to the off-set axis becomes:

(a) <u>When $B_{ij} \neq 0$</u>

$$w_s = \sum_{i=1}^{8} (\omega_i + y_o \phi_i) W_i \sin\delta$$

$$\theta_s = w_{s,y} = \sum_{i=1}^{8} \phi_i W_i \sin\delta \tag{48}$$

$$v_s = \sum_{i=1}^{8} (\rho_i - z_o\phi_i)W_i \sin\delta$$

$$u_s = \sum_{i=1}^{8} [\eta_i - z_o(\frac{m\pi}{a})\omega_i - y_o(\frac{m\pi}{a})\rho_i]W_i \cos\delta \qquad (48)$$

or, written in matrix form,

$$\begin{Bmatrix} w_s \\ \theta_s \\ v_s \\ u_s \end{Bmatrix} \begin{bmatrix} & X_1 & \\ & 4 \times 8 & \end{bmatrix} \begin{Bmatrix} W_1 \\ \vdots \\ W_8 \end{Bmatrix} \qquad (49)$$

i.e.,

$$\{d_{PS}\} = [X_1]\{R_1\} \qquad (50)$$

where $\{d_{PS}\}$ and $\{R_1\}$ are self-evident.

(b) <u>When $B_{ij} = 0$</u>

$$w_s = \sum_{i=1}^{4} (\omega_{wi} + y_o\phi_{wi})W_i \sin\delta$$

$$\theta_s = w_{s,y} = \sum_{i=1}^{4} \phi_{wi}W_i \sin\delta \qquad (51)$$

$$v_s = \sum_{i=1}^{4} (\rho_{ui}U_i - z_o\phi_{wi}W_i)\sin\delta$$

$$u_s = \sum_{i=1}^{4} [\eta_{ui} U_i - z_o(\frac{m\pi}{a})\omega_{wi} W_i - y_o(\frac{m\pi}{a})\rho_{ui} U_i]\cos\delta \tag{51}$$

Or, written in matrix form,

$$\begin{Bmatrix} w_s \\ \theta_s \\ v_s \\ u_s \end{Bmatrix} = \begin{bmatrix} & & \\ & X_1^* & \\ & (4 \times 8) & \\ & & \end{bmatrix} \begin{Bmatrix} W_1 \\ \vdots \\ W_4 \\ U_1 \\ \vdots \\ U_4 \end{Bmatrix} \tag{52}$$

i.e.,

$$\{d_{PS}\} = [X_1^*]\{R_1^*\} \tag{53}$$

where $\{R_1^*\}$ is self-evident.

In a similar manner Equation (45) giving the forces of the flat plate element with respect to the off-set axis becomes:

(a) <u>When $B_{ij} \neq 0$</u>

$$\begin{aligned}
(M_{22})_s &= \sum_{i=1}^{8} [(m_{22})_i + y_o q_i - z_o(n_{22})_i] W_i \sin\delta \\
(Q)_s &= \sum_{i=1}^{8} [q_i + z_o(n_{12})_i(\frac{m\pi}{a})] W_i \sin\delta \\
(N_{12})_s &= \sum_{i=1}^{8} (n_{12})_i W_i \cos\delta \\
(N_{22})_s &= \sum_{i=1}^{8} [(n_{22})_i + y_o(n_{12})_i(\frac{m\pi}{a})] W_i \sin\delta
\end{aligned} \tag{54}$$

Or, written in matrix form,

$$\begin{Bmatrix} (M_{22})_s \\ Q_s \\ (N_{12})_s \\ (N_{22})_s \end{Bmatrix} = \begin{bmatrix} & & \\ & X_2 & \\ & 4 \times 8 & \\ & & \end{bmatrix} \begin{Bmatrix} W_1 \\ \vdots \\ W_8 \end{Bmatrix} \tag{55}$$

i.e.,

$$\{f_{PS}\} = [X_2]\{R_1\} \tag{56}$$

where $\{f_{PS}\}$ is self-evident.

(b) When $B_{ij} = 0$

$$\begin{aligned}
(M_{22})_s &= \sum_{i=1}^{4} [(m_{22})_{wi} W_i + y_o q_{wi} W_i - z_o (n_{22})_{ui} U_i] \sin\delta \\
(Q)_s &= \sum_{i=1}^{4} [q_{wi} W_i + z_o (n_{12})_{ui} (\frac{m\pi}{a}) U_i] \sin\delta \\
(N_{12})_s &= \sum_{i=1}^{4} (n_{12})_{ui} U_i \cos\delta \\
(N_{22})_s &= \sum_{i=1}^{4} [(n_{22})_{ui} + y_o (n_{12})_{ui} (\frac{m\pi}{a})] U_i \sin\delta
\end{aligned} \tag{57}$$

Or, written in matrix form:

$$\begin{Bmatrix} (M_{22})_s \\ Q_s \\ (N_{12})_s \\ (N_{22})_s \end{Bmatrix} = \begin{bmatrix} & & \\ & X_2^* & \\ & (4 \times 8) & \\ & & \end{bmatrix} \begin{Bmatrix} W_1 \\ W_4 \\ U_1 \\ U_4 \end{Bmatrix} \tag{58}$$

i.e.,

$$\{f_{PS}\} = [X_2^*]\{R_1^*\} \tag{59}$$

Thus, when $B_{ij} \neq 0$, Equations (50) and (56) and when $B_{ij} = 0$, Equations (53) and (59) give the displacements and forces of the flat plate element along the off-set axis (Figure 2.7) and with respect to the off-set local axes, x_s, y_s, and z_s. Since the angle between the elements is arbitrary, it is convenient to transform these to a global coordinate system so that consistent displacements and forces can be matched for continuity and equilibrium. Figure 2.8 shows the neutral plane AB of a flat plate element. For clarity the local x axis in the neutral plane and the parallel x_s axis at the off-set points are not shown. X, Y, Z are the global axes. ψ is the angle measured positive in the clockwise direction from the global Y axis to the local y axis.

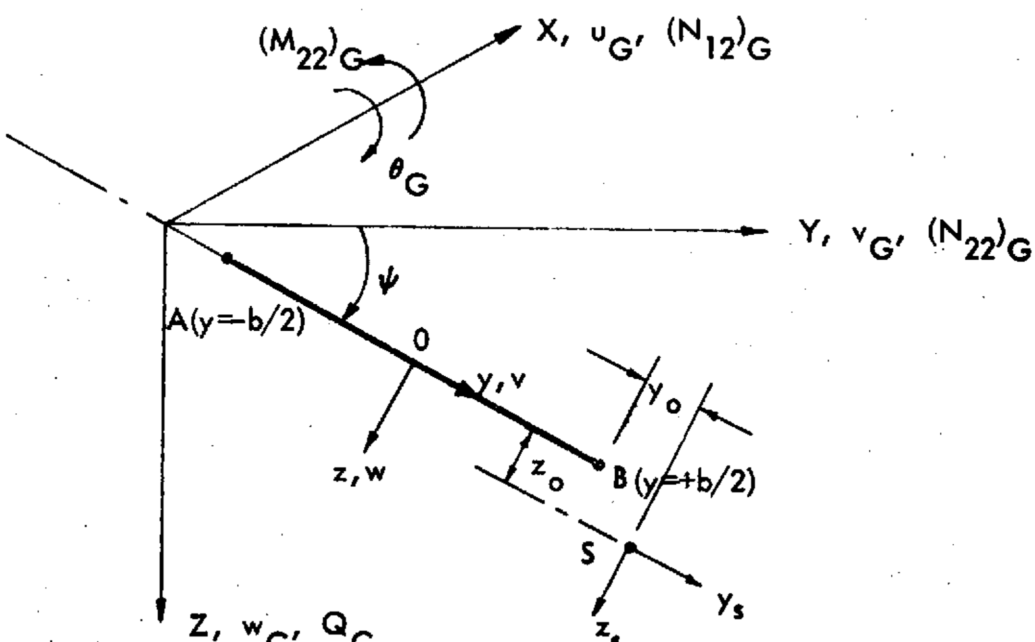


FIGURE 2.8. Global Coordinate System for Flat Plate Elements

Subscript G is used to identify the quantities referred to the global axes. Their positive directions are as indicated in Figure 2.8.

The four displacements of Equation (44) on transformation to the global axes become:

$$\begin{Bmatrix} w_G \\ \theta_G \\ v_G \\ u_G \end{Bmatrix} = \begin{bmatrix} \cos\psi & 0 & \sin\psi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\psi & 0 & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} w_s \\ \theta_s \\ v_s \\ u_s \end{Bmatrix} \qquad (60)$$

Or, written in contracted form:

$$\{d_{PG}\} = [T_d]\{d_{PS}\} \qquad (61)$$

In the following superscripts + and - are used with the various matrix and vector designations to differentiate the corresponding quantities along the sides $y = +b/2$ and $y = -b/2$, respectively. For the side $y = +b/2$, thus, $\{d_{PG}\}$, $[X_1]$, etc., are designated $\{d_{PG}^{+}\}$, $[X_1^{+}]$, etc. Similarly, $\{d_{PG}^{-}\}$, $[X_1^{-}]$, etc., refer to the side $y = -b/2$.

When $B_{ij} \neq 0$, substitution of Equation (50) in Equation (61) yields:

$$\begin{aligned} \{d_{PG}^{+}\} &= [T_d][X_1^{+}]\{R_1\} \\ &= [X_3^{+}]\{R_1\} \end{aligned} \qquad (y = +b/2) \qquad (62)$$

$$\begin{aligned} \{d_{PG}^{-}\} &= [T_d][X_1^{-}]\{R_1\} \\ &= [X_3^{-}]\{R_1\} \end{aligned} \qquad (y = -b/2) \qquad (63)$$

Similarly when $B_{ij} = 0$, substitution of Equation (53) in Equation (61) yields:

$$\{d_{PG}^+\} = [T_d][X_1^{*+}]\{R_1^*\} \qquad (y = +b/2) \qquad (64)$$
$$= [X_3^{*+}]\{R_1^*\}$$

$$\{d_{PG}^-\} = [T_d][X_1^{*-}]\{R_1^*\} \qquad (y = -b/2) \qquad (65)$$
$$= [X_3^{*-}]\{R_1^*\}$$

In a similar manner, the forces in the local coordinates given by Equation (56) when $B_{ij} \neq 0$ and by Equation (59) when $B_{ij} = 0$, can be transformed to the global axes. Thus, when $B_{ij} \neq 0$:

$$\{f_{PG}^+\} = [T_f][X_2^+]\{R_1\} \qquad (y = +b/2) \qquad (66)$$
$$= [X_4^+]\{R_1\}$$

$$\{f_{PG}^-\} = [-T_f][X_2^-]\{R_1\} \qquad (y = -b/2) \qquad (67)$$
$$= [X_4^-]\{R_1\}$$

and when $B_{ij} = 0$:

$$\{f_{PG}^+\} = [T_f][X_2^{*+}]\{R_1^*\} \qquad (y = +b/2) \qquad (68)$$
$$= [X_4^{*+}]\{R_1^*\}$$

$$\{f_{PG}^-\} = [-T_f][X_2^{*-}]\{R_1^*\} \qquad (y = -b/2) \qquad (69)$$
$$= [X_4^{*-}]\{R_1^*\}$$

In the above equations:

$$\{f_{PG}\} = \begin{Bmatrix} (M_{22})_G \\ Q_G \\ (N_{12})_G \\ (N_{22})_G \end{Bmatrix} \tag{70}$$

and

$$[T_f] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\psi & 0 & \sin\psi \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\psi & 0 & \cos\psi \end{bmatrix} \tag{71}$$

Also, $[-T_f]$ indicates the transformation matrix $[T_f]$, (Equation (71)), with the signs of all elements reversed. This is necessitated by the sign convention used for the flat plate element forces in the local coordinates, (see Figure 2.4), where the positive forces along the side $y = +b/2$ have directions opposite to the positive forces along the side $y = -b/2$.

When $B_{ij} \neq 0$, Equations (62), (63), (66), and (67) give the displacements and forces of the flat plate element along the off-set axis, with respect to the global axes. When $B_{ij} = 0$, these are given by Equations (64), (65), (68), and (69). All 'X' matrices in these equations are 4 x 8 in size.

2.5 Beam Element

Any straight prismatic component of the structure under uniaxial compression, which behaves as a beam is defined here as a beam element. Typical examples are lumps of boron fiber reinforcements between stiffeners in stiffened plates, beads or lips or stiffeners, corner fillets of extended stiffeners, etc. See Figure 2.9.

2.5.1 Material and Geometry Constants for a Beam Element

Beam elements can in general be layered. Figure 2.10 shows the geometry of two particular types which are treated in some detail in the next section.

For each layer the basic material properties involved are E_{11}^{k}, (the Youngs Modulus in the axial direction) and G_{23}^{k} (the Shear Modulus).

In the case of the rectangular beam all layers are assumed to have the same depth 'b'. The origin of the axes system is for convenience taken at the geometric center of the bead or flange.

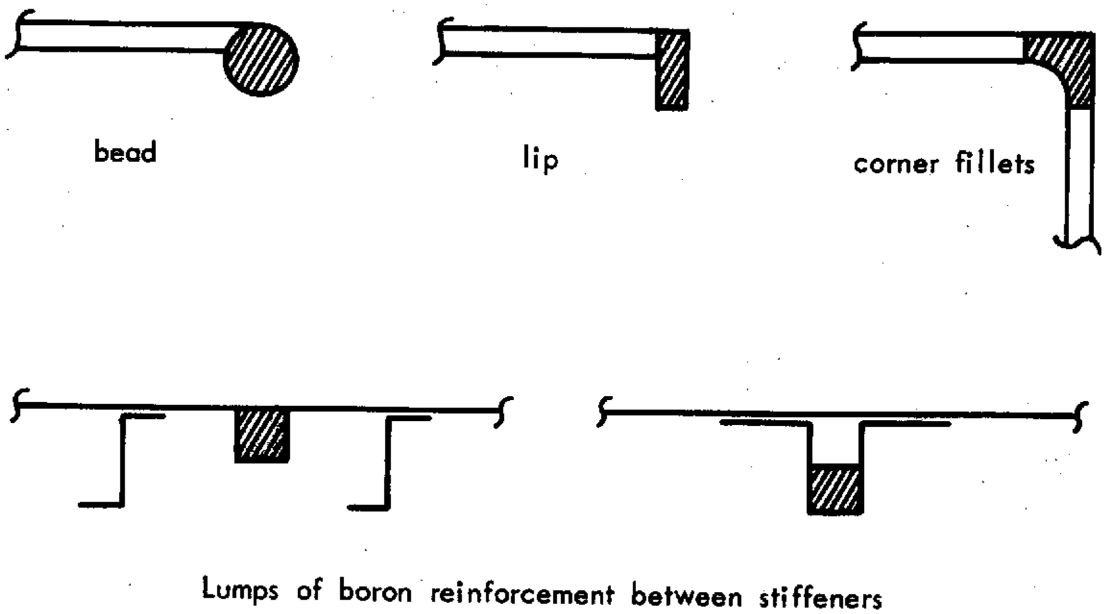


FIGURE 2.9. Typical Examples of Beam Idealization

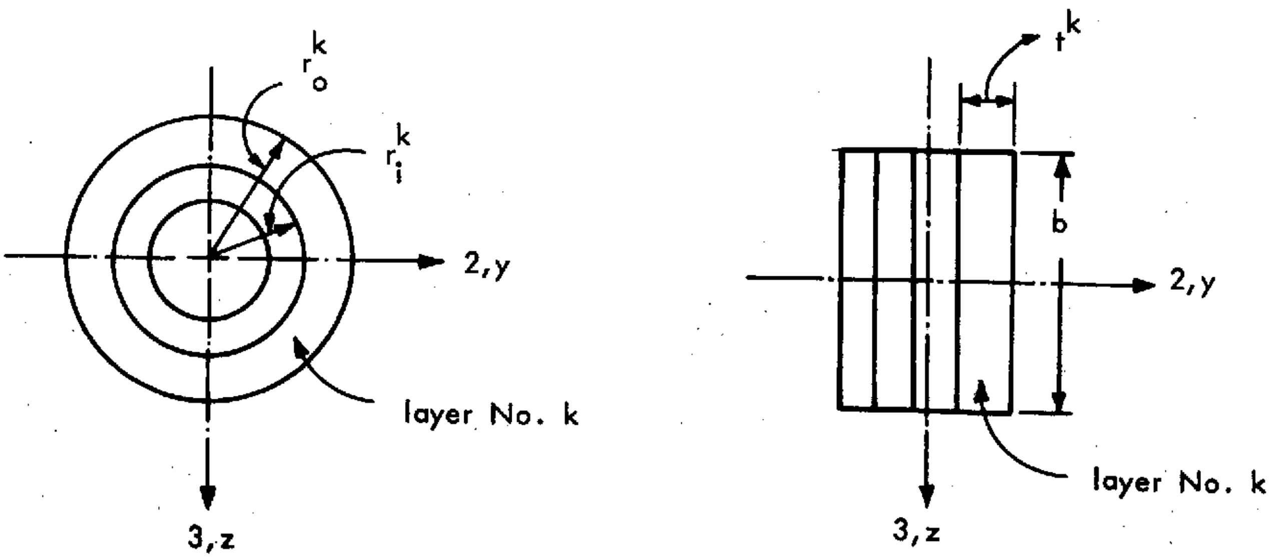


FIGURE 2.10. Circular and Rectangular Layered Beams

2.5.2 Overall Stiffness of a Beam Element

In order to evaluate the forces in the beam element (see Section 2.5.3), the overall stiffnesses (bending stiffness, torsion stiffness, etc.) are needed. With the exception of the St. Venant torsional stiffness ($G_{23}J$) of the layered flange, these are approximated by the sum of the individual layer stiffnesses. Thus:

$$
\begin{aligned}
E_{11} I_{zz} &= \sum_{k=1}^{\ell} E_{11}^{k} I_{zz}^{k} \\
E_{11} I_{yy} &= \sum_{k=1}^{\ell} E_{11}^{k} I_{yy}^{k} \\
E_{11} A_{b} &= \sum_{k=1}^{\ell} E_{11}^{k} A_{b}^{k} \\
E_{11} \Gamma &= \sum_{k=1}^{\ell} E_{11}^{k} \Gamma^{k} \\
\bar{\sigma} I_{p} &= \sum_{k=1}^{\ell} \bar{\sigma}^{k} I_{p}^{k}
\end{aligned}
\qquad (72)
$$

$\bar{\sigma}^k$ is the compressive stress in the k^{th} layer caused by the external loading, the axial strain over the entire cross-section of the structure under consideration being constant. See Section 2.8.

For the particular types of layered beams shown in Figure 2.10, the various geometric properties involved in Equation (72) are detailed in Table 2.1. It is readily seen that when all the layers of the rectangular beam are horizontal (vertical layers are shown in Figure 2.10) it is sufficient to interchange I_{zz}^k and I_{yy}^k in Equation (72). The St. Verant torsional stiffness ($G_{23}J$) of the layered circular beam is approximated by:

TABLE 2.1. Geometric Constants for Circular and Rectangular Layered Beams

Geometric Property of k^{th} Layer	Circular Beam	Rectangular Beam
Area: A_b^k	$\pi \left\{ (r_o^k)^2 - (r_i^k)^2 \right\}$	$b^k t^k$
Moment of Inertias: I_{zz}^k	$\frac{\pi}{4} \left\{ (r_o^k)^4 - (r_i^k)^4 \right\}$	$\frac{b^k (t^k)^3}{12} + A_b^k \left[\left\{ \left(\sum_{k=1}^{k} t^k \right) - \frac{t^k}{2} \right\} - \left\{ \frac{\sum_{k=1}^{\ell} t^k}{2} \right\} \right]^2$
I_{yy}^k	----- " -----	$\frac{t^k (b^k)^3}{12}$
Polar Moment of Inertia: I_p^k	----- " -----	$I_{zz}^k + I_{yy}^k$
Warping Constant: Γ^k	0	$\frac{1}{144} (b^k t^k)^3$ **

**Argyris, J. H., and Dunne, P. C., "Handbook of Aeronautics No. 1: Structural Principles and Data, Part 2," p. 122 and 126, Fourth Edition, The New Era Publishing Co., London.

$$G_{23}J = \sum_{k=1}^{\ell} G_{23}^{k}J^{k}$$

where (73)

$$J^{k} = I_{zz}^{k} + I_{yy}^{k}$$

For layered rectangular beams an overall G_{23} value is evaluated from the individual layer G_{23}^{k} values as:

$$G_{23} = \frac{\sum_{k=1}^{\ell} G_{23}^{k}A_{b}^{k}}{\sum_{k=1}^{\ell} A_{b}^{k}} \tag{74}$$

The overall torsional stiffness $G_{23}J$ is then approximated using J based on overall dimensions of the flange. Thus,

$$J = \frac{bt^{3}}{16}\left[\frac{16}{3} - 3.36\,\frac{t}{b}\cdot\left\{1 - \frac{1}{12}\left(\frac{t}{b}\right)^{4}\right\}\right]^{**} \tag{75}$$

(NOTE: When $t/b > 1$ interchange t and b.)

where

$$t = \sum_{k=1}^{\ell} t^{k}$$

**Argyris, J. H., and Dunne, P. C., "Handbook of Aeronautics No. 1: Structural Principles and Data, Part 2," pps. 122 and 126, Fourth Edition, The New Era Publishing Co., London.

2.5.3 Equations for a Beam Element

The basic equations for bending of beams and torsion of beams, including the effect of axial compression, are used to calculate the forces, due to buckling displacements, along the length of the beam element. These displacements and forces are shown in Figures. 2.11 and 2.12.

The equations given in this section for forces, are with respect to the geometric center of the beam element.

The total applied end load $\overline{P}_b$ in the layered beam element is given by:

$$\overline{P}_b = \sum_{k=1}^{\ell} \bar{\sigma}^k A_b^k \tag{76}$$

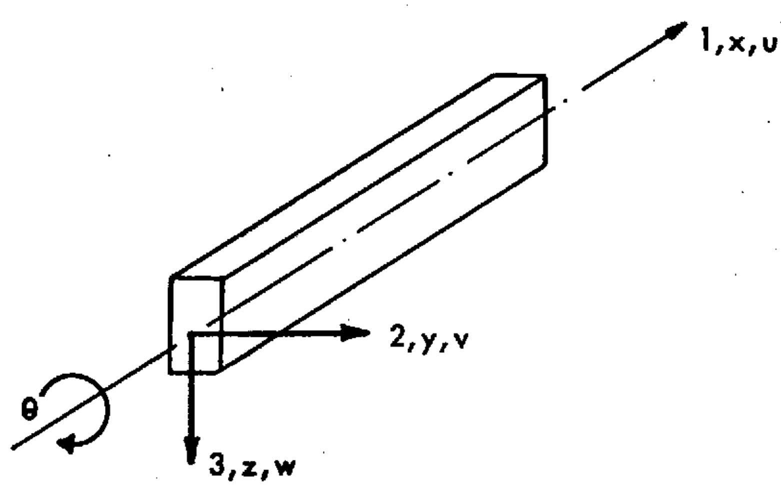


Figure 2.11. Displacements of Beam Element

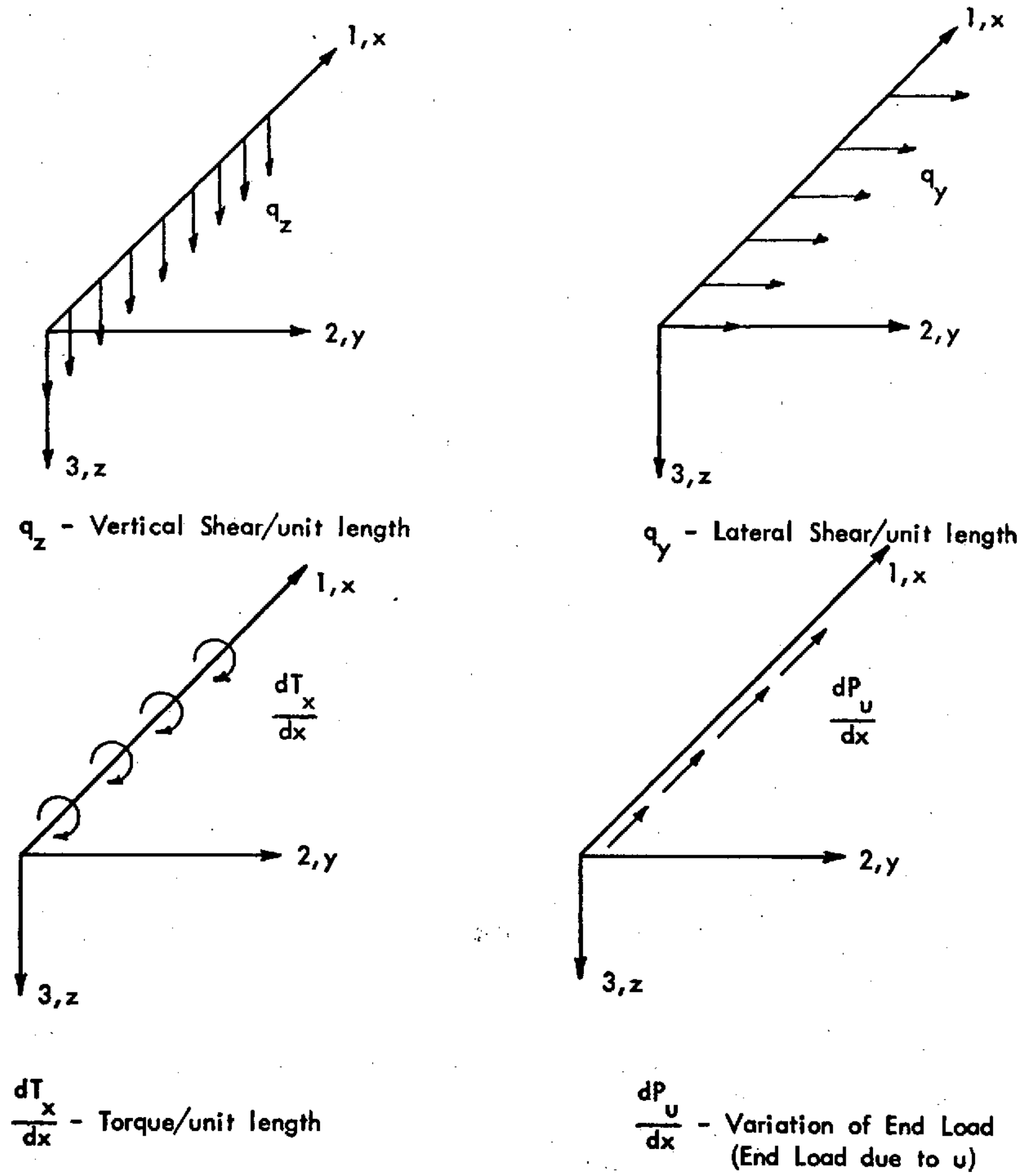


Figure 2.12. Forces on Beam Element

The expressions for the vertical shear q_z and the lateral shear q_y (see Figure 2.12) are obtained from beam theory (shear effects ignored). Refer: Timoshenko, S. P., Gere, J. M., "Theory of Elastic Stability," McGraw-Hill, 2nd Edition, 1961, p. 2.

$$q_z = E_{11}\, I_{yy} \frac{d^4 w}{dx^4} + \bar{P}_b \cdot \frac{d^2 w}{dx^2}$$

$$q_y = E_{11}\, I_{zz} \frac{d^4 v}{dx^4} + \bar{P}_b \cdot \frac{d^2 v}{dx^2} \qquad (77)$$

The expression for the torque dT_x/dx is obtained from torsion theory of beams, including warping effects. Refer: Argyris, J. H., Dunne, P. C., Handbook of Aeronautics No. 1: Structural Principles and Data, Part II, New Era Publishing Co., London, 4th Edition, p. 140.

$$\frac{dT_x}{dx} = (E_{11}\, \Gamma \frac{d^4\theta}{dx^4} - G_{23}\, J \frac{d^2\theta}{dx^2} + \bar{\sigma}\, I_p \frac{d^2\theta}{dx^2}) \qquad (78)$$

θ is the twist of the beam element. For a circular beam, the first and the last terms on the right-hand side of Equation (78) are zero. For a rectangular beam the effect of these terms are small. However, they are retained as the program has the capability to allow for any type of beam, by giving Γ, J, and I_p as inputs.

Axial displacement (u), of the beam element, caused by the buckling deformation gives rise to end load P_u. The rate of change of this end load is given by:

$$\frac{dP_u}{dx} = -E_{11}\, A_b \cdot \frac{d^2 u}{dx^2} \qquad (79)$$

The following, general buckling displacement functions are assumed for the beam element:

$$
\begin{aligned}
w &= W \sin \delta \\
v &= V \sin \delta \\
u &= U \cos \delta \\
\theta &= \Theta \sin \delta
\end{aligned}
\tag{80}
$$

where

$$\delta = \frac{m\pi x}{a}$$

These functions automatically satisfy the simply supported boundary conditions along the edges $x = 0$ or a, where the external applied load acts.

For a given level of axial load, P_b, in the beam element, the forces (Figure 2.12) due to the buckling displacements of Equation (80) can be readily calculated from Equations (77) to (79). These forces are with respect to the local axes, x, y, z.

2.5.4 Forces and Displacements of Beam Elements

In this section the forces in the beam elements due to the buckling displacements of Equation (80) are initially evaluated at the geometric center with respect to the local axes. See Figure 2.12.

As for the flat plate elements in Section 2.4.4, these displacements and forces to be used for inter-element continuity and equilibrium, are then transferred to an off-set axis, before finally transforming them to the global axes system.

The beam element displacements involved are:

$$w, \quad \theta, \quad u, \quad v \tag{81}$$

The beam element forces involved are:

$$q_z, \quad \frac{dT_x}{dx}, \quad \frac{dP_u}{dx}, \quad q_y \tag{82}$$

Equations (77), (78), and (79) of Section 2.5.3 give the forces referred to an axes system at the geometric center of beam element section. The displacements are also referred to the same axes. See Figures 2.11 and 2.12.

By substitution, all quantities are expressed in terms of the displacements, Equation (80). Thus, from Equations (77) to (79):

$$q_z = [E_{11}\, I_{yy} \left(\frac{m\pi}{a}\right)^4 - \bar{P}_b \cdot \left(\frac{m\pi}{a}\right)^2] \cdot W \cdot \sin\delta = W\, \xi_1 \sin\delta$$

$$q_y = [E_{11}\, I_{zz} \left(\frac{m\pi}{a}\right)^4 - \bar{P}_b \cdot \left(\frac{m\pi}{a}\right)^2 \cdot V \cdot \sin\delta = V\, \xi_4 \sin\delta \tag{83}$$

$$\frac{dT_x}{dx} = [E_{11}\, \Gamma \left(\frac{m\pi}{a}\right)^4 + G_{12}\, J \left(\frac{m\pi}{a}\right)^2 - \sigma\, I_p \left(\frac{m\pi}{a}\right)^2] \cdot \Theta \sin\delta = \Theta\, \xi_2 \sin\delta$$

$$\frac{dP_u}{dx} = [\, E_{11}\, A_b \left(\frac{m\pi}{a}\right)^2]U \cos\delta = U\, \xi_3 \cos\delta$$

In satisfying the inter-element considerations, it is necessary to appropriately transfer these beam element forces and displacements to the line (parallel to x-axis) along which compatibility is enforced.

Let, the off-sets to this line measured positive in the positive directions from the origin of the beam element be y_o and z_o. See Figure 2.13.

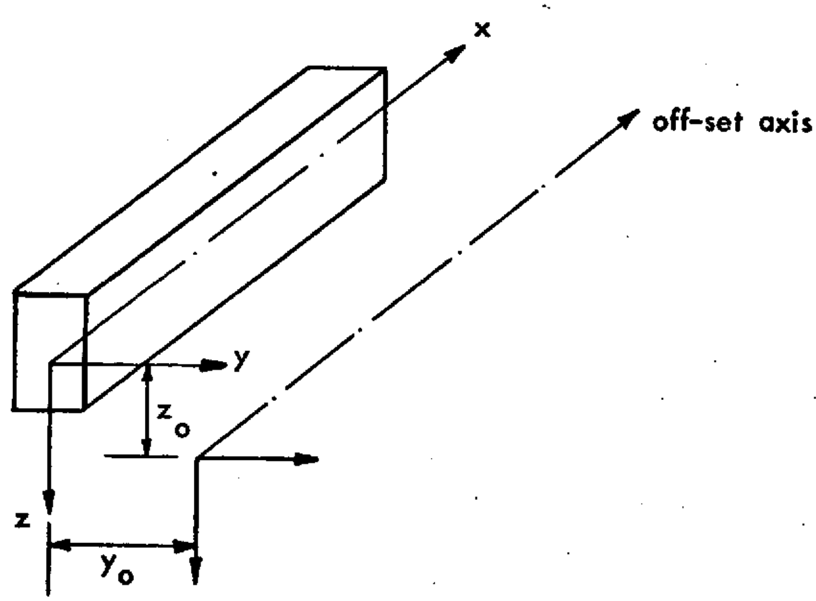


Figure 2.13. <u>Beam Element Off-Sets</u>

Using the subscript 's' to denote the quantities with respect to the shifted axes system, the displacements of the beam element become:

$$
\begin{aligned}
w_s &= w + y_o \cdot \theta \\
\theta_s &= \theta \\
v_s &= v - z_o \cdot \theta \\
u_s &= u - z_o \cdot w_{,x} - y_o \cdot v_{,x}
\end{aligned}
\tag{84}
$$

After substitution from Equation (80), the above equations become:

$$
\begin{Bmatrix} w_s \\ \theta_s \\ v_s \\ u_s \end{Bmatrix} = \begin{bmatrix} 0 & 0 & \sin\delta & y_o \sin\delta \\ 0 & 0 & 0 & \sin\delta \\ 0 & \sin\delta & 0 & -z_o \sin\delta \\ \cos\delta & [-y_o(\frac{m\pi}{a})\cos\delta] & [-z_o(\frac{m\pi}{a})\cos\delta] & 0 \end{bmatrix} \begin{Bmatrix} U \\ V \\ W \\ \Theta \end{Bmatrix} \tag{85}
$$

Or, written in a contracted form:

$$
\{d_{BS}\} = [X_5]\{R_2\} \tag{86}
$$

where $[X_5]$ is a 4 x 4 matrix.

Similarly, the forces on the beam element are transferred to the off-set axes as:

$$
\left(\frac{dT_x}{dx}\right)_s = \frac{dT_x}{dx} + q_y \cdot z_o - q_z \cdot y_o
$$

$$(q_z)_s = q_z + \frac{d^2P_u}{dx^2} \cdot z_o$$

$$(\frac{dP_u}{dx})_s = \frac{dP_u}{dx} \tag{87}$$

$$(q_y)_s = q_y + \frac{d^2P_u}{dx^2} \cdot y_o$$

On substitution from (83) the above equations become:

$$\begin{Bmatrix} (\frac{dT_x}{dx})_s \\ (q_z)_s \\ (\frac{dP_u}{dx})_s \\ (q_y)_s \end{Bmatrix} \begin{bmatrix} 0 & [z_o\xi_4 \sin\delta] & [-y_o\xi_1 \sin\delta] & [\xi_2 \sin\delta] \\ [-z_o\xi_3(\frac{m\pi}{a})\sin\delta] & 0 & [\xi_1 \sin\delta] & 0 \\ [\xi_3 \cos\delta] & 0 & 0 & 0 \\ [-y_o\xi_3(\frac{m\pi}{a})\sin\delta] & [\xi_4 \sin\delta] & 0 & 0 \end{bmatrix} \begin{Bmatrix} U \\ V \\ W \\ \theta \end{Bmatrix} \tag{88}$$

Or, written in the contracted form:

$$\{f_{BS}\} = [X_6]\{R_2\} \tag{89}$$

$[X_6]$ is a 4 x 4 matrix and its elements are functions of the external load $\overline{P}_b$ on the beam element.

Equations (86) and (89) give the displacements and forces of the beam element with respect to the local axes x_s, y_s, z_s (Figure 2.13). Their positive directions are the same as those shown in Figure 2.12.

As done for the flat plate element, in Section 2.4.4, these displacements and forces are now transformed to the global axes. The positive directions of displacements and forces with respect to the global axes are the same as in Figure 2.8, where the angle ψ, is now the angle between the global Y axis and the local y axis of the beam element, measured positive in the clockwise direction.

Using subscript G to denote the quantities with respect to the global axes, the beam element displacements given by Equation (86) are transformed as:

$$\{d_{BG}\} = [T_d]\{d_{BS}\} = [T_d][X_5]\{R_2\} = [X_7]\{R_2\} \tag{90}$$

where

$$\{d_{BG}\} = \begin{Bmatrix} w_G \\ \theta_G \\ v_G \\ u_G \end{Bmatrix} \tag{91}$$

The transformation matrix $[T_d]$ is the same as in Equation (61). $[X_7]$ is a 4 x 4 matrix.

In a similar manner the beam element forces given by Equation (89) are transformed to the global axes as:

$$\{f_{BG}\} = [T_f]\{f_{BS}\} = [T_f][X_6]\{R_2\} = [X_8]\{R_2\} \tag{92}$$

where

$$\{f_{BG}\} = \begin{Bmatrix} (M_{22})_G \\ Q_G \\ (N_{12})_G \\ (N_{22})_G \end{Bmatrix} \tag{93}$$

The transformation matrix $[T_f]$ is the same as in Equation (71). $[X_8]$ is a 4 x 4 matrix.

Thus Equations (90) and (92) give the displacements and forces of beam elements along the off-set axis, with respect to the global axes.

2.6 Inter-Element Displacement Continuity and Force Equilibrium

In this section, the displacements and forces of the flat plate and beam elements derived in Section 2.4.4 and 2.5.4, with respect to the global axes, are used to illustrate the principle of enforcing inter-element continuity and equilibrium.

Figure 2.14 shows a typical junction of three flat plate elements (1), (3), and (4) and a beam element (2). The dashed line is the outer contour of each element. The neutral plane of each element is indicated by the continuous line. Point B is the geometric center of the beam element. The angle ψ is measured positive in the clockwise direction from the global Y axis to the local y axis of each element.

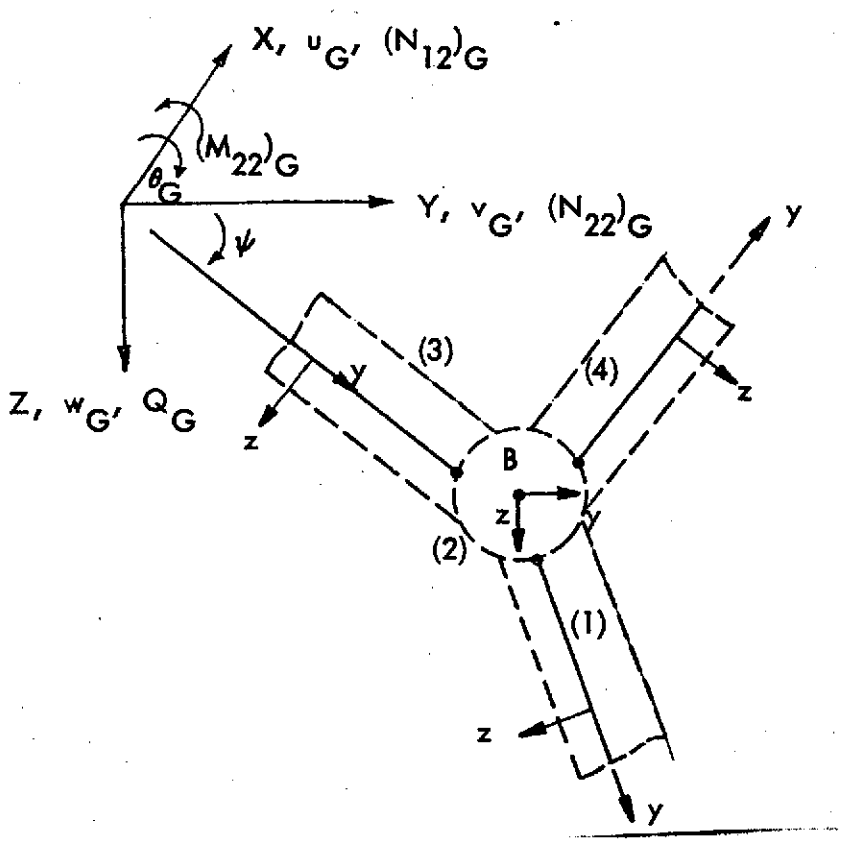


FIGURE 2.14. A Typical Junction of Elements

For the purpose of illustration, the axial line through B is chosen for enforcing inter-element continuity and equilibrium. Consequently the beam element has zero off-sets. In the program, whenever beam elements are encountered the identical line through the beam element is chosen for inter-element matching, thereby having to input only the off-sets for the flat plate elements.

Table 2.2 shows the forces and displacements involved in the inter-element matching and the corresponding equation numbers, for the typical junction shown in Figure 2.14.

Since there are no external loads or constraints at the junction, the force equilibrium at B yields:

$$\{f_{PG}^-\}_{(1)} + \{f_{BG}\}_{(2)} + \{f_{PG}^+\}_{(3)} + \{f_{PG}^-\}_{(4)} = 0 \tag{94}$$

and inter-element continuity yields:

$$\{d_{PG}^-\}_{(1)} = \{d_{BG}\}_{(2)} = \{d_{PG}^+\}_{(3)} = \{d_{PG}^-\}_{(4)} \tag{95}$$

The subscripts (1), (2), (3), and (4) denote the element numbers as shown in Figure 2.14. Using the equations shown in Table 2.2, Equations (94) and (95) can be written in matrix form, assuming $B_{ij} \neq 0$ for all flat plate elements, as:

$$\begin{bmatrix} [X_4^-]_{(1)} & [X_8]_{(2)} & [X_4^+]_{(3)} & [X_4^-]_{(4)} \\ [X_3^-]_{(1)} & -[X_7]_{(2)} & & \\ & [X_7]_{(2)} & -[X_3^+]_{(3)} & \\ & & [X_3^+]_{(3)} & -[X_3^-]_{(4)} \end{bmatrix} \begin{Bmatrix} \{R_1\}_{(1)} \\ \{R_2\}_{(2)} \\ \{R_1\}_{(3)} \\ \{R_1\}_{(4)} \end{Bmatrix} = 0 \tag{96}$$

TABLE 2.2.

Forces and Displacements for Inter-Element Matching at the Junction in Figure 2.14

Element No.	Force and Displacement Expression	Equation Number	
		$B_{ij} \neq 0$	$B_{ij} = 0$
(1) & (4)	$\{d_{PG}^{-}\}$	(63)	(65)
	$\{f_{PG}^{-}\}$	(67)	(69)
(3)	$\{d_{PG}^{+}\}$	(62)	(64)
	$\{f_{PG}^{+}\}$	(66)	(68)
(2)	$\{f_{BG}\}$	(92)	
	$\{d_{BG}\}$	(90)	

Similar equations can be readily written when one or more flat plate elements have $B_{ij} = 0$. A junction of elements could also be elastically restrained, clamped or simply supported.

These variations are not included in the present program. The reference quoted in Section 2.0 gives the analysis when the junction is elastically restrained.

2.7 Boundary Conditions along any Unloaded Edge of Flat Plate Element

Any unloaded edge of a flat plate element can, in general, be free, clamped, or simply supported.

The forces and displacements associated with these boundary conditions are those with respect to the neutral plane of the flat plate element. By putting $\psi = y_o = z_o = 0$, in the appropriate equations of Section 2.4.4, the forces and displacements at either side of the flat plate element are readily obtained.

(a) Free Edge

The forces along a free edge are zero. Hence $\{f_{PG}^{+}\} = 0$, or $\{f_{PG}^{-}\} = 0$. Thus, when $B_{ij} \neq 0$, Equations (66) and (67) yield:

$$[X_4^{+}]\{R_1\} \Big|_{\substack{\psi = y_o = z_o = 0 \\ y = +b/2}} = 0 \qquad \text{or} \qquad [X_4^{-}]\{R_1\} \Big|_{\substack{\psi = y_o = z_o = 0 \\ y = -b/2}} = 0 \tag{97}$$

Similarly, when $B_{ij} = 0$, Equations (68) and (69) yield:

$$[X_4^{*+}]\{R_1^{*}\} \Big|_{\substack{\psi = y_o = z_o = 0 \\ y = +b/2}} = 0 \qquad \text{or} \qquad [X_4^{*-}]\{R_1^{*}\} \Big|_{\substack{\psi = y_o = z_o = 0 \\ y = -b/2}} = 0 \tag{98}$$

(b) Clamped Edge

The boundary conditions are:

(i) $w = 0$

(ii) $w_{,y} = 0$

(iii) $u = 0$ or $N_{12} = 0$ (99)

(iv) $N_{22} = 0$ or $v = 0$

For conditions (iii) and (iv), the first of each are used in the program. In the above equations, when $B_{ij} \neq 0$, substitution from Equations (28) to (30) and Equation (35) results in

$$[X_9^{+}]\{R_1\}\Big|_{y=+b/2} = 0 \qquad \text{or} \qquad [X_9^{-}]\{R_1\}\Big|_{y=-b/2} = 0 \tag{100}$$

Similarly, when $B_{ij} = 0$, substitution from Equations (36) to (38) and Equation (43) results in:

$$[X_9^{*+}]\{R_1^*\}\Big|_{y=+b/2} = 0 \qquad \text{or} \qquad [X_9^{*-}]\{R_1^*\}\Big|_{y=-b/2} = 0 \tag{101}$$

(c) Simply Supported Edge

The classical simple support conditions are $w = M_{22} = u = N_{22} = 0$. When $B_{ij} \neq 0$, substitution from Equations (28), (33), (30), and (35) results in:

$$[X_{10}^{+}]\{R_1\}\Big|_{y=+b/2} = 0 \qquad \text{or} \qquad [X_{10}^{-}]\{R_1\}\Big|_{y=-b/2} = 0 \tag{102}$$

Similarly, when $B_{ij} = 0$, substitution from Equations (36), (41), (38), and (43) results in:

$$[X_{10}^{*+}]\{R_1^*\}\Big|_{y=+b/2} = 0 \qquad \text{or} \qquad [X_{10}^{*-}]\{R_1^*\}\Big|_{y=-b/2} = 0 \tag{103}$$

The unloaded edges of a flat plate element can also be elastically restrained. This is, however, not included in the present program. The analysis for this case is given in the reference quoted in Section 2.0.

2.8 Buckling Load and Buckled Form of the Structure under Uniaxial Compression

The principle of inter-element matching discussed in Section 2.6 and the equations for the boundary conditions along any unloaded side of the flat plate element derived in Section 2.7, are applied in this section, to determine the uniaxial compressive buckling load of the structure. For easy reference, the basic equations used are collected in Table 2.3. It is pointed out that all the displacements and forces referred to are those due to the buckling deformation.

The first example considered is the arbitrary structure shown in Figure 2.1. The edge $y = -b/2$ of element (1) is assumed to be simply supported and the edge $y = +b/2$ of the element (7) is assumed to be clamped. Assuming $B_{ij} \neq 0$ for all flat plate elements, considerations of the above boundary conditions and those of inter-element matching, using Table 2.3, result in the following equations:

$$\begin{array}{c}
\begin{matrix} (1) & (2) & (3) & (4) & (5) & (6) & (7) & (8) \end{matrix} \\
\left[\begin{array}{cccccccc}
X_{10}^- & & & & & & & \\
X_3^+ & -X_3^- & & & & & & \\
X_4^+ & X_4^- & & & & & & \\
 & X_3^+ & -X_7 & & & & & \\
 & & X_7 & -X_3^- & & & & \\
 & X_4^+ & X_8 & X_4^- & & & & \\
 & & & X_3^+ & -X_3^- & & & \\
 & & & X_4^+ & X_4^- & & & \\
 & & & & X_3^+ & -X_3^- & & \\
 & & & & & X_3^- & -X_3^- & \\
 & & & & X_4^+ & X_4^- & X_4^- & \\
 & & & & & X_3^+ & & -X_7 \\
 & & & & & X_4^+ & & X_8 \\
 & & & & & & X_9^+ &
\end{array}\right]
\end{array}
\begin{Bmatrix} R_{1(1)} \\ R_{1(2)} \\ R_{2(3)} \\ R_{1(4)} \\ R_{1(5)} \\ R_{1(6)} \\ R_{1(7)} \\ R_{2(8)} \end{Bmatrix} = 0 \qquad (104)$$

TABLE 2.3

Summary of Flat Plate Element and Beam Element Equations Used in Buckling Equations

Force or Displacement	Equation	Equation No.	B_{ij}	Size of Matrix [X]	Remarks
$\{d_{PG}^{\pm}\}$	$[X_3^{\pm}]\{R_1\}$ $[X_3^{*\pm}]\{R_1^*\}$	(62), (63) (64), (65)	$B_{ij} \neq 0$ $B_{ij} = 0$	4 x 8 4 x 8	Flat plate element displacement
$\{f_{PG}^{\pm}\}$	$[X_4^{\pm}]\{R_1\}$ $[X_4^{*\pm}]\{R_1^*\}$	(66), (67) (68), (69)	$B_{ij} \neq 0$ $B_{ij} = 0$	4 x 8 4 x 8	Flat plate element forces
$\{d_{BG}\}$	$[X_7]\{R_2\}$	(90)	-	4 x 4	Beam element displacements
$\{f_{BG}\}$	$[X_8]\{R_2\}$	(92)	-	4 x 4	Beam element forces
-	$[X_4^{\pm}]\{R_1\} = 0$ $[X_4^{*\pm}]\{R_1^*\} = 0$	(97) (98)	$B_{ij} \neq 0$ $B_{ij} = 0$	4 x 8 4 x 8	Free edge along the unloaded side of the flat plate element
-	$[X_9^{\pm}]\{R_1\} = 0$ $[X_9^{*\pm}]\{R_1^*\} = 0$	(100) (101)	$B_{ij} \neq 0$ $B_{ij} = 0$	4 x 8 4 x 8	Clamped along the unloaded side of the flat plate element
-	$[X_{10}^{\pm}]\{R_1\} = 0$ $[X_{10}^{*\pm}]\{R_1^*\} = 0$	(102) (103)	$B_{ij} \neq 0$ $B_{ij} = 0$	4 x 8 4 x 8	Simply supported along the unloaded side of the flat plate element

The numbers in parenthesis are the element numbers, the same as in Figure 2.1.

Or, written in contracted form:

$$[X_B]\{R_B\} = 0 \tag{105}$$

A similar equation can be easily written when $B_{ij} = 0$ for one or more flat plate elements.

The matrix $[X_B]$ is square and in this case of size 56 x 56. In general $[X_B]$ is n x n where

n = (8 x number of flat plate elements + 4 x number of beam elements) .

A nontrivial solution of Equation (105) is obtained when the determinant

$$|X_B| = 0 \tag{106}$$

$|X_B|$ is called the "buckling determinant" of the structure. It can be readily verified that a common factor of $\sin\delta$ or $\cos\delta$ where $\delta = m\pi x/a$ can be taken out of each row of $|X_B|$ and ignored.

The various terms of the "buckling determinant" involve, in addition to the geometric and material properties of flat plate elements and beam elements:

(a) Axial half-wave number m which is assumed to be the same for all elements.

(b) When $B_{ij} \neq 0$ the p_i values for each flat plate element from Equation (18), or when $B_{ij} = 0$, the p_{ui} and p_{wi} values from Equations (23) and (25). These roots are, in general, a function of the external load level $\overline{N}_{11}$ in each flat plate element.

(c) The external load level $\overline{P}_b$ in each beam element.

The load level in each element of the structure as discussed above, is determined on the basis of uniform axial strain. In the present program, a load level $(\bar{N}_{11})_{(j)}$ is assumed on the first flat plate element, (j) denoting the element number. The first flat plate element need not be the first element in the structure. The corresponding axial strain ϵ_{11}^{o} is evaluated from:

$$\epsilon_{11}^{o} = \frac{(\bar{N}_{11})_{(j)}}{\left(\sum_{k=1}^{\ell} \frac{t^k}{S_{11}^k}\right)_{(j)}} \tag{107}$$

The load level in any other flat plate element (n) corresponding to this strain is then obtained from:

$$(\bar{N}_{11})_{(n)} = \epsilon_{11}^{o} \left(\sum_{k=1}^{\ell} \frac{t^k}{S_{11}^k}\right)_{(n)} \tag{108}$$

Similarly, the applied axial compressive stress $\bar{\sigma}^k$, corresponding to the same axial strain ϵ_{11}^{o}, in any layer k of the beam element is:

$$\bar{\sigma}^k = \epsilon_{11}^{o} \cdot E_{11}^k \tag{109}$$

Knowing the load per unit width $\bar{N}_{11}$ on all flat plate elements and the stress $\bar{\sigma}^k$ in all layers of the beam elements, the total load on the structure can be readily evaluated.

As mentioned earlier, the various terms in the "buckling determinant," $|X_B|$ are functions of the external load level. Hence the buckling load is obtained from Equation (106), by iteration. An axial mode m (the same for all the elements) and an initial load level $(N_{11})_{(j)}$ in the first flat plate element is assumed. The load levels in all the other elements are evaluated. Corresponding to these load levels for each flat plate elements, when $B_{ij} \neq 0$, Equation (18) is solved for the p_i values or when $B_{ij} = 0$, Equations (23) and (25) are solved for p_{ui} and p_{wi} values. The "buckling determinant" $|X_B|$ is formed and evaluated. If it is nonzero, the above procedure is repeated in steps, increasing the load level at each step, until lowest axial load at which the "buckling determinant" vanishes is determined. This is, then, the buckling load for the assumed axial mode m. The initial assumed load level must be sufficiently low, so as to ensure that the lowest buckling load is not missed, for any assumed m. The buckled shape of the cross-section as determined from the eigenvector (discussed later in this section), might indicate whether the buckling load determined is the lowest one or not. Buckling loads for a series of axial modes are evaluated and the minimum determined.

Double roots from Equation (18) or Equations (23) and (25) cause two columns of the matrix $[X_B]$ in Equation (105) to be identical. This difficulty is overcome by identifying within close limits, the load level yielding double roots and ignoring this small load interval. The roots when complex, occur in conjugate pairs. Using the conjugate pair of roots in $[X_B]$ results in "conjugate columns."

For the purpose of determining the buckling load, each pair of "conjugate columns" in the "buckling determinant" can be converted into a column of only real numbers and a column of only imaginary numbers, by a process of addition and subtraction. By taking a common factor $i(=\sqrt{-1})$ outside, for each column of imaginary numbers, the buckling determinant is made to contain only real numbers.

Further, it can be readily shown that in Equation (105) those elements of the vector $\{R_B\}$ corresponding to a pair of "conjugate columns" in $[X_B]$, will be conjugates, too. This property is made use of in solving for the eigenvector corresponding to the buckling load, in order to evaluate the buckled form of the cross-section. By multiplying out and adding like terms in each equation of Equation (104), it can be rewritten such that the new matrix corresponding to $[X_B]$ and the new vector corresponding to $\{R_B\}$ contain only real terms. Each pair of conjugate elements of the original vector $\{R_B\}$ will be replaced by its real part and its imaginary part. The new system of equations containing only real terms, are solved to obtain the modified vector $\{R_B\}$, by inverse iteration as indicated in Appendix A. The original vector $\{R_B\}$ of Equation (105) can be readily obtained from this. The buckled form of the cross-section, then follows, from the displacement equations of Section 2.4.4 and 2.5.3.

A second example considered is that of a hat-section stiffened plate shown in Figure 2.2. The stiffener dimensions and their spacing is repetitive. In the figure, the numbers in parentheses are the element numbers and the dots indicate the inter-element junctions. The two unloaded sides of the panel are assumed to be simply supported. The equations for inter-element continuity and equilibrium, together with those for the boundary conditions along the unloaded sides of the stiffened plate, can be written in a manner similar to Equation (104). Repetitive nature of the stiffeners in this example is taken advantage of in writing these equations. The equations for the stiffened plate can be considered to consist of three basic parts; namely,

(i) A set of equations, designated as $[T_a]\{R_a\}$, for the left side of the stiffened plate, representing the boundary conditions along the first dot and the inter-element conditions along the next five dots in Figure 2.2 involving elements (1) to (7). The dots are counted from the left side of the stiffened plate.

(ii) A second set of equations, designated as $[T_b]\{R_b\}$, for a repetitive unit, representing the inter-element conditions along the next six dots (i.e., dots 7 to 12, both inclusive), involving elements (7) to (14). The number of repetitions of this repetitive unit, in this example, will be one less than the total number of stiffeners, since the first stiffener has already been taken into account under (i).

(iii) A third set of equations, designated as $[T_c]\{R_c\}$, for the right side of the stiffened plate, representing the conditions along the last two dots, involving elements (28) and (29).

A repetitive unit consists of a certain number of inter-element junctions, together with any unloaded edges, which repeat themselves. Using Table 2.3, these equations can be written as:

$$
[T_a]\{R_a\} = \begin{array}{c}
\begin{array}{ccccccc} (1) & (2) & (3) & (4) & (5) & (6) & (7) \end{array} \\
\left[\begin{array}{c|c|c|c|c|c|c}
X_{10}^- & & & & & & \\
X_3^+ & -X_3^- & & & & & \\
X_4^+ & X_4^- & & & & & \\
 & X_3^+ & -X_3^- & & & & \\
 & & X_3^- & -X_3^- & & & \\
 & X_4^+ & X_4^- & X_4^- & & & \\
 & & X_3^+ & & -X_3^- & & \\
 & & X_4^+ & & X_4^- & & \\
 & & & X_3^+ & & -X_3^- & \\
 & & & & & X_3^- & -X_3^- \\
 & & & X_4^+ & & X_4^- & X_4^- \\
 & & & & X_3^+ & -X_3^- & \\
 & & & & X_4^+ & X_4^+ &
\end{array}\right]
\end{array}
\left\{\begin{array}{c} R_{1(1)} \\ \hline R_{1(2)} \\ \hline R_{1(3)} \\ \hline R_{1(4)} \\ \hline R_{1(5)} \\ \hline R_{1(6)} \\ \hline R_{1(7)} \end{array}\right\} \qquad (110)
$$

$$
[T_b]\{R_b\} =
\begin{array}{c}
\begin{array}{cccccccc}
\scriptstyle(7) & \scriptstyle(8) & \scriptstyle(9) & \scriptstyle(10) & \scriptstyle(11) & \scriptstyle(12) & \scriptstyle(13) & \scriptstyle(14)
\end{array} \\
\left[\begin{array}{c:c:c:c:c:c:c:c}
X_3^{+} & -X_3^{-} & & & & & & \\
X_4^{+} & X_4^{-} & & & & & & \\
 & X_3^{+} & -X_3^{-} & & & & & \\
 & X_4^{+} & X_4^{-} & & & & & \\
 & & X_3^{+} & -X_3^{-} & & & & \\
 & & & X_3^{-} & -X_3^{-} & & & \\
 & & X_4^{+} & X_4^{-} & X_4^{-} & & & \\
 & & & X_3^{+} & & -X_3^{-} & & \\
 & & & X_4^{+} & & X_4^{-} & & \\
 & & & & X_3^{+} & & -X_3^{-} & \\
 & & & & & & X_3^{-} & -X_3^{-} \\
 & & & & X_4^{+} & & X_4^{-} & X_4^{-} \\
 & & & & & X_3^{+} & -X_3^{+} & \\
 & & & & & X_4^{+} & X_4^{+} &
\end{array}\right]
\end{array}
\left\{\begin{array}{c}
R_{1(7)} \\ \hdashline
R_{1(8)} \\ \hdashline
R_{1(9)} \\ \hdashline
R_{1(10)} \\ \hdashline
R_{1(11)} \\ \hdashline
R_{1(12)} \\ \hdashline
R_{1(13)} \\ \hdashline
R_{1(14)}
\end{array}\right\}
\qquad (111)
$$

$$
[T_c]\{R_c\} = \left[\begin{array}{c:c}
X_3^{+} & -X_3^{-} \\ \hdashline
X_4^{+} & X_4^{-} \\ \hdashline
 & X_{10}^{+}
\end{array}\right]
\left\{\begin{array}{c}
R_{1(28)} \\ \hdashline
R_{1(29)}
\end{array}\right\}
\qquad (112)
$$

Equation (110), appropriate number of repetitions of Equation (111) and Equation (112) together yield the complete set of equations for the hat-stiffened plate. Since there are elements common to adjacent units, like elements (7), (14), (28), etc., it is easy to see that the vectors $\{R_a\}$, $\{R_b\}$, and $\{R_c\}$ will be overlapping each other with common terms. Thus for the four stiffener case of Figure 2.2, the equations can be written in the following form:

$$\begin{bmatrix} T_a & & & & \\ & T_b & & & \\ & & T_b & & \\ & & & T_b & \\ & & & & T_c \end{bmatrix} \begin{Bmatrix} R_a \\ R_b \\ R_b \\ R_b \\ R_c \end{Bmatrix} = 0 \qquad (113)$$

(113)

Or, written in a contracted form:

$$[X_B]\{R_B\} = 0 \qquad (114)$$

This is the same as Equation (105) of the first example.

The two dotted lines between R_a, R_b, etc., in Equation (113) indicates the overlap between them as mentioned earlier. $[X_B]$ is a square matrix. The determination of the critical load and the corresponding buckled form of the cross-section, from Equation (114) follows identical lines as described for Equation (105).

It is seen that the matrix $[X_B]$ is banded and contains repetitive submatrices $[T_b]$. This property is taken advantage of in evaluating the buckling determinant $|X_B|$ for critical load evaluation. The method is discussed in Appendix A.

Buckling load of any other structure under uniaxial compression can be determined in a similar manner. As mentioned above, the present program assumes the "buckling determinant" $|X_B|$ to be banded. This precludes any structure where the first and last elements are inter-connected, in which case the banded property is lost.

It is important to note that the buckling loads and the corresponding eigen-modes are determined from a general instability analysis, in that no restrictions are placed on the buckling deformation of the cross-section (except that the angles between elements remain unchanged). The eigen-modes are indicative of overall or local nature of instability. In contrast the classical buckling analysis assumes restricted deformation of the cross-section as in flexural (Euler) mode, torsional mode, local mode, etc. Such simplifying restrictions can sometimes result in missing the lowest buckling load.

Attention is also drawn to the fact that the loaded edge of each element making up the structure is assumed to be simply supported. Thus each flat plate element has a line condition of simple support and each beam element a point condition of simple support at the loaded edges. Thus for structures of complex cross-sectional shape, the overall end conditions in the present analysis will be different compared to the conventional Euler instability theory, where the structure as a whole is idealized to a line member and simply supported, resulting in a point condition of simple support at the loaded ends. The effect of this will be small when the axial half-wave length of buckling is small compared to the length of the structure.

Finally, the discussion in Section 2.3 regarding the beam idealization is once again emphasized.

2.9 Differences in Matrix Designations between Theory and Program

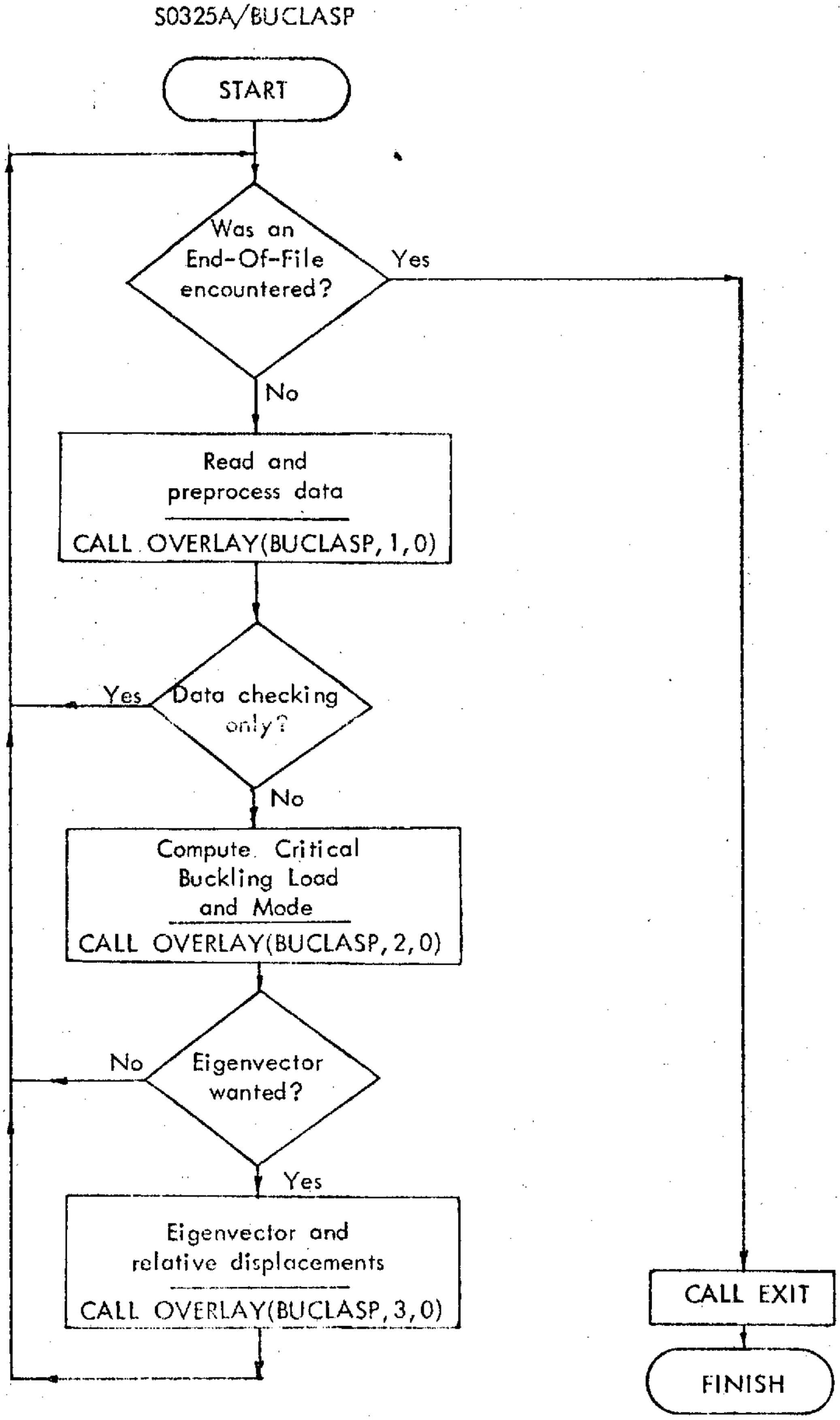
As shown in Section 2.8, for any structure of uniform cross-section and under uniaxial compression, the final buckling equation (e.g., Equations (105) and (114)), is systematically built-up from the flat plate and beam element equations summarized in Table 2.3. The [X] matrices in this table are designated in the program with type numbers. Table 2.4 shows the one to one correspondence between the [X] matrices and the type numbers. Note the minus sign in front of some of matrices.

TABLE 2.4

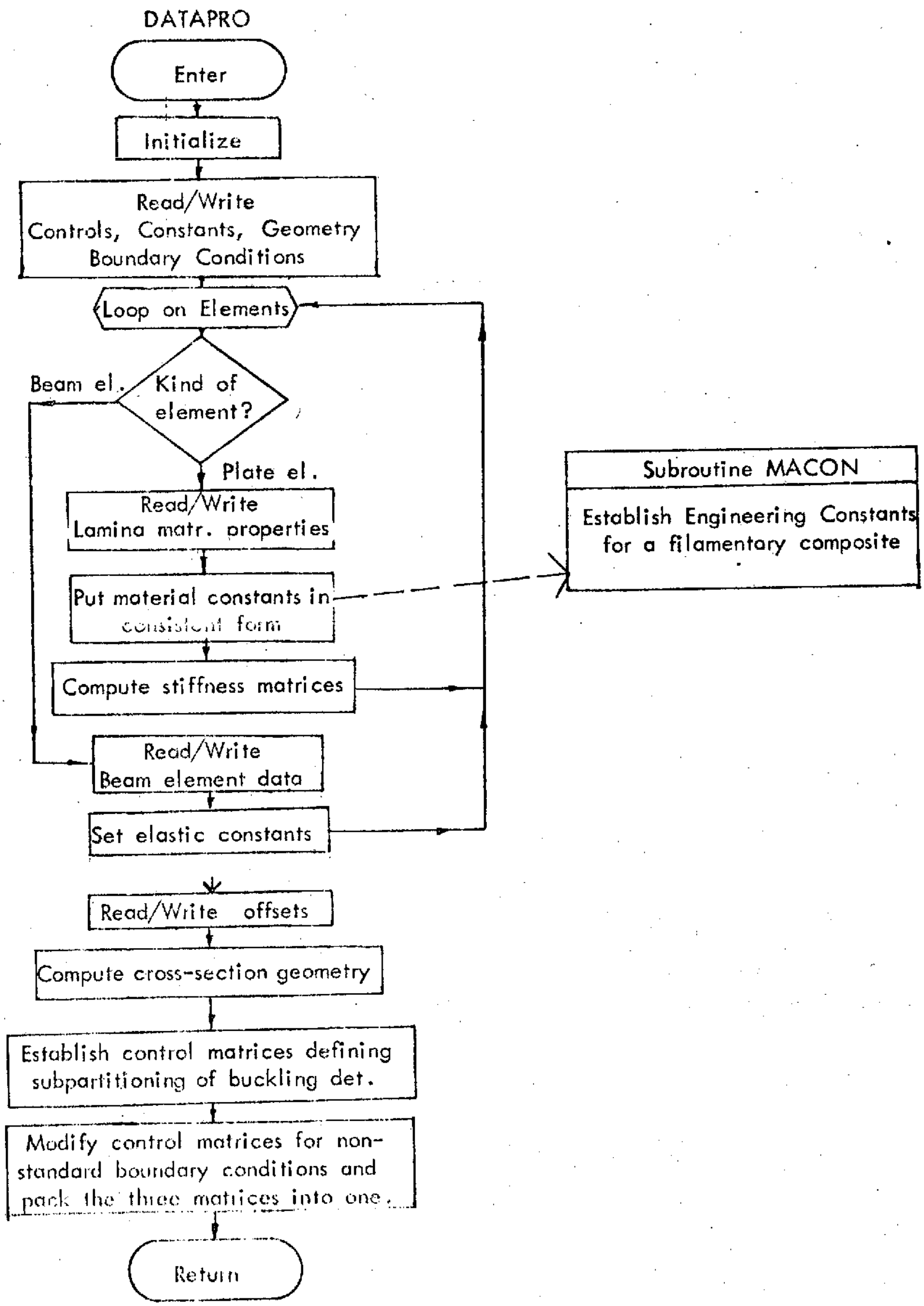
Matrix Designations in the Theory and the Corresponding Matrix Type Numbers Used in the Program

Matrix Designation in the Theory (From Table 2.3)	Matrix Type Number in the Program
$[X_3^{+}]$, $[X_3^{*+}]$	3
$-[X_3^{-}]$, $-[X_3^{*-}]$	1
$[X_4^{+}]$, $[X_4^{*+}]$	4
$[X_4^{-}]$, $[X_4^{*-}]$	2
$[X_7]$	7
$-[X_7]$	9
$[X_8]$	8, 10
$[X_9^{+}]$, $[X_9^{*+}]$	14
$[X_9^{-}]$, $[X_9^{*-}]$	12
$[X_{10}^{+}]$, $[X_{10}^{*+}]$	13
$[X_{10}^{-}]$, $[X_{10}^{*-}]$	11

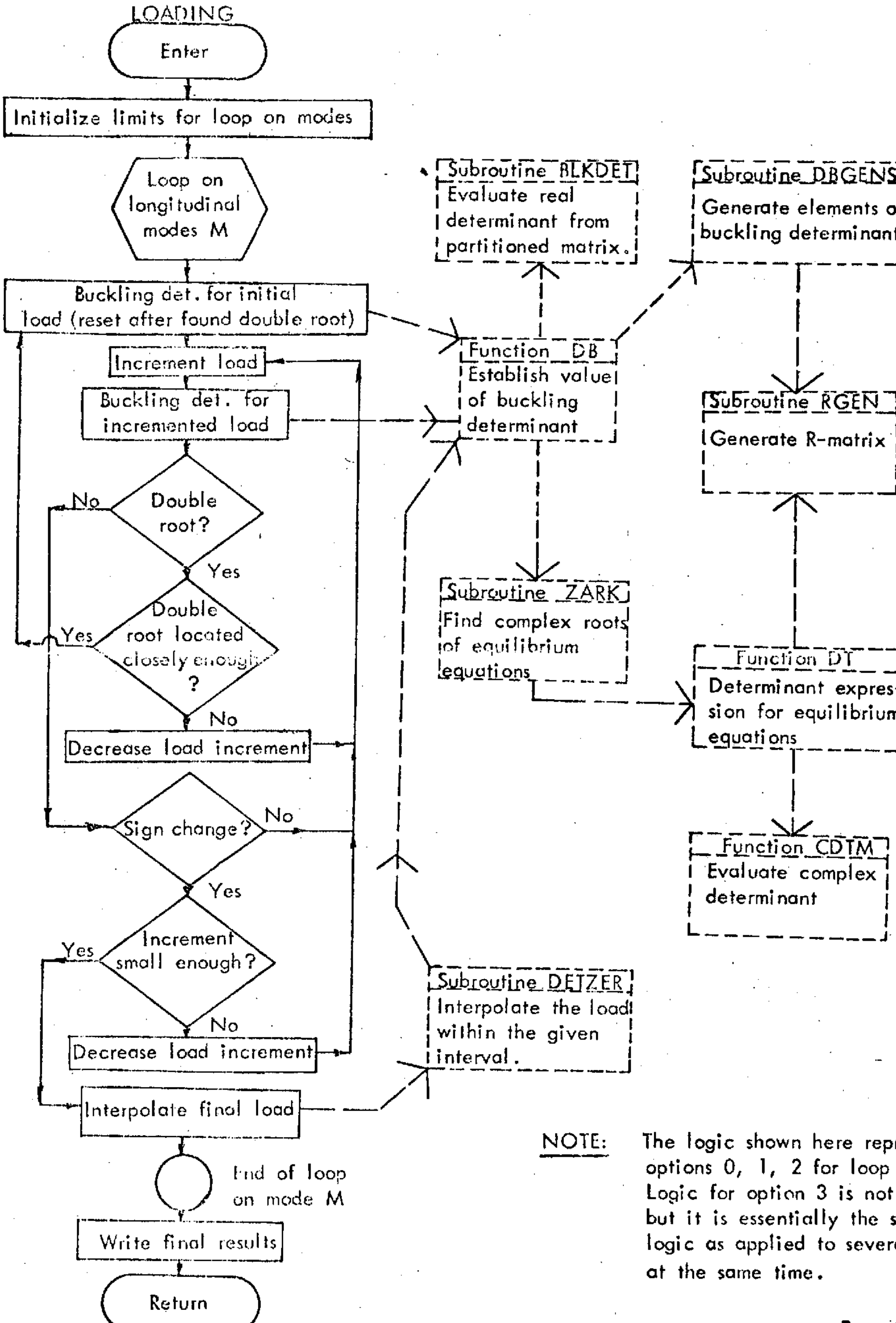
3.0 OVERALL LOGICAL FLOW OF PROGRAM



3.1 Logical Flow of Overlay for Data Preprocessing

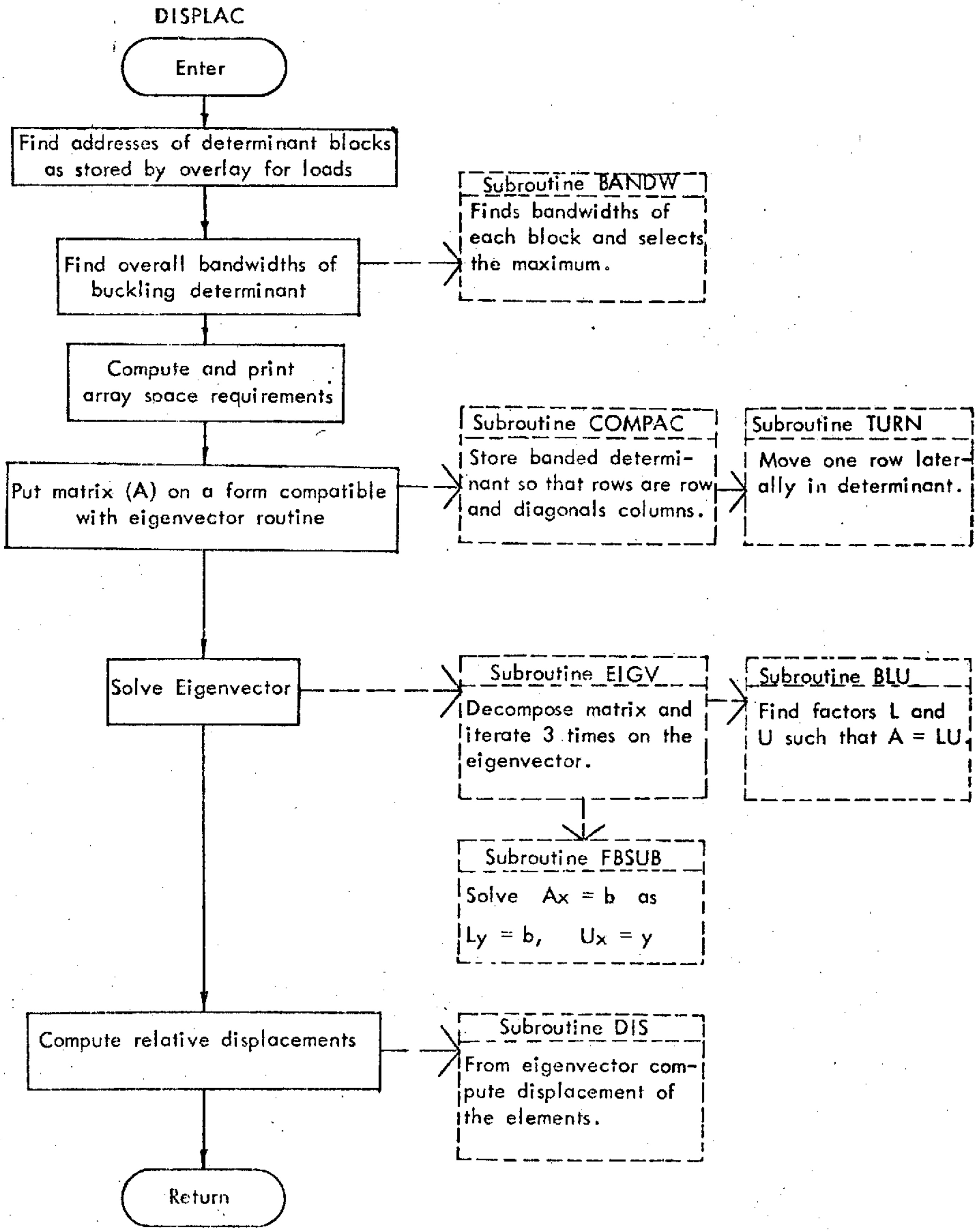


3.2 Logical Flow of Overlay for Load Solution



NOTE: The logic shown here represents options 0, 1, 2 for loop on modes. Logic for option 3 is not shown but it is essentially the same logic as applied to several modes at the same time.

3.3 Logical Flow of Overlay for Eigenvector Solution



4.0 COMPUTER PROGRAM USAGE

This section describes the program from a standpoint of usage and describes the required control cards as well as the requirements to field length and timing. Specifications are also given for the card preparation.

4.1 Machine Requirements

This program is developed using the CDC 6600 computers at the Boeing Company in Renton, Washington, and requires peripheral equipment as follows:

a. Card reader
b. Line printer
c. Tape drive (if program is supplied on tape)
d. Disk - scratch storage space

The program has been written with the intent of compatibility with the CDC 6600 computer installation at NASA, Langley Research Center.

4.2 Operating System

The operating system used is the Boeing version of SCOPE 3.1. The program is written in FORTRAN IV and for the sake of easy conversion no special features of the Boeing Company compUter software have been used.

4.3 Timing and Output Estimates

Time consumption for one data set depends on various factors:

a. Number of modes that are investigated for each data set.

b. Number of plate elements and beam elements required to build up the cross-section The size of the buckling determinant is proportional to total number of elements and for cases with large number of element a large part of the computer time is used for determinant evaluation.

c. Number of plate elements of the same type, in the sense that their plate stiffnesses are the same. For plate elements of the same type the solution of the equilibrium equations will be the same and therefore a time saving occurs.

d. Number of plate elements which are symmetric laminates, for instance isotropic elements. For symmetric laminates two of the four roots of the equilibrium equations are independent of the trial load for the same mode, and this is taken advantage of as these two equations are not resolved for each load.

e. Choice of option for the loop on the modes m. A time saving will take place when the option is used where the loop on modes is done for each trial load and apparent noncritical modes are eliminated from the search as early as possible.

f. A guess value of the load must be provided which is smaller than the actual critical load to be found. If this starting value is close to the critical load a time saving would occur.

The actual computer time consumption for some of the test cases run for program checkout are quoted in the following table (TABLE 4.3.1) and this information can be used for arriving at a time estimate.

In general for single data set runs, and with the intermediate printout switch off, the program will not generate printed output in excess of the default option line limit of 10000_8 (4096 decimal). However, if many data sets are run this line limit can easily be reached and thus it is recommended that the line limit be increased to 100000_8 or 200000_8 during compilation (See Section 3.6), or in the load card.

The average run with one data set, and 2 modes requires from 30 to 100 pages of print, when the intermediate output is suppressed.

If intermediate output is desired, the amount of output will vary considerably according to the relation between startload, location of double roots in the DT-determinant, and the critical load, but the amount is so large the intermediate printout feature is not recommended used except for trouble shooting.

TABLE 4.3.1 Computer Time Consumption - Load Evaluation

Data Set	Order of Buckling Det.	No. of Modes	Total CP Time (sec.)	CP Time (sec.) for Critical Mode	No. of trial loads for critical mode	CP-time per trial load (sec.)
Test 5	170	1	94.0	92.2	67	1.37
Test 2A	216	4	289.	85.2	51	1.67
Test 2B	216	4	533.8	170.0	98	1.73
Test 4A	200	3	70.4	18.6	16	1.16
Test 4B	104	4	46.4	10.5	25	0.42
Test 6A	200	1	41.6	40.5	49	0.83
Test 8	288	1	71.0	68.	36	1.89
Test 3A	24	5	15.1	1.6	12	0.13
Test 3B	24	5	55.9	11.5	78	0.15
Test 3C	20	6	9.6	1.2	17	0.07

4.4 Programmed Diagnostic Messages

MESSAGE:	COMMENT:
Main Program LOADING	
1. Above are 50 tries without change in sign. Double load increment and start over again.	Arrays are getting filled up, so we reset their contents and index to zero. Doubling of load increment increases speed in obtaining the critical load.
2. Limit for number of DB-calls of 400 is exceeded, extrapolate for load. Note answer is not reliable. It is recommended that you review your data for a possible change of start load and load intervals.	Self-explanatory. We have to stop somewhere. Stop computation on this mode, and continue to the next one, if any.
3. Warning - a double root in p - investigate the load region up to this double root.	Self-explanatory. Appears only when intermediate print is switched on.
4. The first load examined after passing the double real roots did not produce significant difference between the roots. Perturb load and try again.	Self-explanatory. Subsequent load is increased by a small amoant.
5. The startload was too close to double real roots. Perturb load and try again.	Self-explanatory. Subsequent load is increased by small amount.

MESSAGE:	COMMENT:
Subroutine Function DB	
1. ZARK failed to converge in the maximum number of iterations specified.	The complex rootfinder ZARK returned an error code. Increase maximum number of iterations given in program for ZARK. (300 presently) Change startload.
2. ZARK failed - a zero in the path of a subsequent one.	The complex rootfinder ZARK returned an error code. See Section 3.8 of Program Description Document.. Change startload.
3. An error appeared in the P-values. A complex root that is not one of a conjugate pair.	The DT-determinant in polynomial form comes out in only even powers of P (the roots). The roots of this polynomial must then contain only real roots and conjugate pairs.

NOTE:

The comments above in the DB-routine related to the ZARK routine does not appear in normal use of the program, as they are incompatible with the theory. However, if they should appear, see Section 3.8 of Program Description Document.

Subroutine BLKDET:	
1. The matrix A (B or C) has a zero row.	See Section 3.6 of Program Description Document for explanation of this routine.
2. Matrix sizes or relative positions are incompatible.	
3. Zero determinant at block X.	

4.5 Restrictions

The following restrictions apply to this version of the program.

4.5.1 Analysis Oriented Restrictions

a. The loading is uniaxial compression, with uniform strain across the cross-section.

b. Each layer is orthotropic with respect to plate axes.

c. Material is fully elastic.

d. The cross-section is uniform in the axial direction.

e. Loaded edges of each element forming the cross-section are simply supported.

4.5.2 Programming Oriented Restrictions

For restrictions pertaining to the mathematical routines, see attached descriptions of these (ZARK, BLKDET, CDTM, DETZER, EIGV, BLU, FBSUB).

a. Maximum number of elements required to describe the three blocks is 25, and the maximum number of nodes is 20. Maximum number of elements in any block is 14.

b. Maximum number of types of plate elements is 10. These are of the same type in the sense that their stiffness matrices are of the same contents.

c. The maximum number of layers is 25 for plate elements.

d. Maximum number of beam elements is 10.

e. The maximum number of layers is 35 for beam elements.

f. The critical load is located within a load interval os size equal to 1.0% of lower limit of the load interval for loads less than 50 lbs/in. and an interval of 5 lbs/in. for loads larger than 50 lbs/in. before the interpolation routine is used.

g. The tolerance for the interpolation routine DETZER is set to 10^{-8} times the load.

h. If the load increment is less than 0.5% of the load then the spotchecks are not done. Spotcheck means that the last two intervals will be subdivided to check when it is suspected that a critical load is bypassed.

i. If the ratio between slopes of the buckling determinant function for two subsequent intervals is less than 5 then the spotcheck is also abandoned.

j. The double roots of determinant expression det(DT) = 0 are located within a load interval of size equal to 0.1% of the lower limit of the load interval (det(DT) = 0 is the determinant expression for the equilibrium equations). For loads larger than 50 lbs. 0.04% is used.

k. Two real roots of the determinant expression det(DT) = 0 are considered double if they differ by less than 3%.

l. The imaginary part of the complex roots of the determinant expression det(DT) = 0 is set exactly to zero if its numerical value is less than 10^{-6}, or when it is less than 10^{-5} times the real part of the number. A similar test applies to the real part of the number.

m. There is no coupling between bending and stretching when the B-matrix (coupling stiffness) is zero. All the elements of the B-matrix are checked and the matrix is assumed to be zero when all its elements are less than 1.0.

n. Care must be exercised in choosing start load and load step for the iteration process. For instance startload obviously must be smaller than the critical load and load step be small enough to prevent two zero crossings of the buckling determinant in the interval. For the data run to check the program a primary load step of 20 lbs/in. is found to be satisfactory.

o. Maximum number of modes that can be investigated in one data set is 30.

4.6 Operating System Control Cards

4.6.1 Via Source Deck

	Col. 1 on cards
Job Control Record	Sequence Card
	Job Card
	Account Card
	RUN(S,,,,,,200000,,1)
	LGO.
	$^{7}8_{9}$ (end-of-record card)
Program-Source Deck	Source Deck
	$^{7}8_{9}$
Data	*Data Cards
	$6_{7_{8_{9}}}$ (end-of-file card)

4.6.2 Via Relocatable Binary Deck

	Col. 1 on cards
Job Control Record	Sequence Card
	Job Card
	Account Card
	INPUT.
	$^{7}8_{9}$
Binary Decks	Binary Decks
	$^{7}8_{9}$
	$^{7}8_{9}$
Data	*Data Cards
	$6_{7_{8_{9}}}$

---

*NOTE: Repeated data sets do not require End-of-Record cards between them.

4.6.3 Via Absolute Binary Tape

Col. 1 on cards

Job Control Record	Sequence Card
	Job Card
	Account Card
	REQUEST TAPEA. (66-DXXX, MOUNT/INPUT)
	REWIND(TAPEA)
	COPYBF(TAPEA, BUCLASP)
	UNLOAD(TAPEA)
	DROPFIL(TAPEA)
	BUCLASP.
	EXIT.
	UNLOAD(TAPEA)
	7 8 9 (end-of-record card)
Data	*Data
	6 7 8 9 (end-of-file card)

NOTE: The tape number used will correspond to the tape allocated for the program at the installation in question. If the program occupies a file on the tape other than the first the tape should be positioned accordingly.

*NOTE: Repeated data sets to not require End-Of-Record cards between them.

4.6.4 Field Length

The field length estimate required for this program depends upon the data set that is run as the array space for the buckling determinant is allocated in a dynamic manner in blank common. The same is also true for the buckling determinant as stored in a compact banded manner in the overlay for eigenvector solution.

Upon the execution of a particular data set the program will compute and print estimates of the field length required for the load solution as well as the eigenvector solution, something which can be used in later runs.

In Table 4.6.1 below are given field lengths for some of the test cases.

When the eigenvector and relative displacements are also wanted the user must consider if the field length is long enough for the eigenvector overlay to accomodate all the repetitive blocks of the buckling determinant as the full determinant is stored on a compact banded form. For the loading overlay the field length requirement does not increase with an increased number of blocks as only three blocks are stored.

TABLE 4.6.1 Field Lengths

Data Set	Overlay (2.0) Loading	Overlay (3.0) Displacements	Number of Blocks in Buckling Determinant	Governing Overlay	Maximum Number of Blocks in 70K
Test 2A	64400_8	66200_8	7	(3.0)	7
Test 2B	64400_8	66200_8	7	(3.0)	7
Test 3A	52200_8	31000_8	3	(2.0)	26
Test 3B	52200_8	31000_8	3	(2.0)	26
Test 3C	51500_8	30400_8	3	(2.0)	38
Test 4A	64200_8	62300_8	6	(2.0)	7
Test 4B	53100_8	37500_8	6	(2.0)	24

4.7 Input Data Format

The data input to this program consists only of cards, and no data tapes are required.

CARD 1 (8A10)

Cols.

1-80 Title of run. Any characters anywhere on the card. This title is printed out in several strategic places in the output for the purpose of identification.

CARD 2 (16I5) IPC Control Array

Cols.

1-5 IPC(1) = 1 Give print of intermediate results (I).
= blank Suppress intermediate results (I).

These intermediate check results (I) include the roots P of the determinant expression det(DT) = 0 (equilibrium equations).

6-10 IPC(2) = 1 Give print of intermediate results (II).
= blank Suppress intermediate results (II).

Intermediate check results (II) include the DB matrix and its determinant (boundary conditions, buckling determinant).

76-80 IPC(16) = 1 Calculate only elastic constants, lamina stiffness matrix, Q, plate stiffness matrices A, B, D for a case when fiber and matrix properties are given, or if for any other reason only the material constants are required.

CARD 3 (16I5)

Cols.

1-5 JPC(1) Panel type identification number. (See TABLE 4.7.1) Blank for nonstandard types, i.e. when JPC(4) = 1.

6-10 JPC(2) Number of blocks that the buckling determinant is partitioned into. Includes two end blocks plus the repetive blocks. Cases with start block only or startblock and endblock only are permitted. (See Section 3.6 of Program Description Document.) See Section 5.2 regarding checkout of this item.

Cols.

11-15	JPC(3)	= 0	(blank) No relative displacements.
		= 1	Relative displacements will be computed if the available core allows it.
16-20	JPC(4)	= 0	(blank) Buckling determinant type matrices are set up internally.
		= 1	Read in buckling determinant type matrices. See CARD 17 and 18.

CARD 4 (16I5)

Cols.

1-5	MMI		Starting value for the loop on the longitudinal buckling mode M.
6-10	MMA		End value for the loop on the longitudinal buckling mode M.
11-15	MOPT		Option control for the loop on the buckling mode M. Four options exist:
		= 0	or blank, start loop at 1 and loop until a minimum load is found, then interrupt (Max. 30 loops).
		= 1	Start the loop at MMI and loop until a minimum load is found, then interrupt (Max. 30 loops).
		= 2	Start the loop at MMI and loop to MMA regardless of whether a minimum load is found or not.
		= 3	Under this option, the loop on modes is done for each load-step. Instead of finding the critical load for each mode in order to select the smallest load, when a sign change occurs in one or more modes, the program eliminates modes which do not give sign change, as these modes are not critical. Finally, only the critical mode remains and the load is found for this mode. This option should be used when only the buckling load for the critical mode is wanted. Considerable time can be saved in cases when many modes have to be investigated. See Section 5.2 regarding checkout of this option.

NOTE: The arrays are dimensioned so that MMA - MMI $\leqslant$ 30.

CARD 5 (3I10,4F10.2)

Cols.

1-10	LL	Number of plate elements in the section (i.e. in start block, one repetitive block and end block).
11-20	LB	Number of beam elements in the section (i.e. in start block, one repetitive block and end block).
21-30	NOD	Number of nodes in the section (i.e. in start block, one repetitive block and end block).
31-40	AL	Length of the section (inches).
41-50	STLD	Starting load in the search for the critical load (lb/in.). (See note for explanation.)
51-60	SINC	Primary load interval (lb/in.). (See note.)
61-70	SINC2	Secondary load interval (lb/in.). (See note.)

NOTE: The loads STLD, SINC, and SINC2 are given as line loads on the first element of the section. We start with a load equal to STLD and increase it with step of size SINC until a change in the sign of the DB-determinant will occur. Using the last load before the sign change as a new start load, we now increment the load by SINC2 until the zero-crossing is encountered again. From this point on the same procedure is repeated, and the increment halved each time, until the critical load (zero-crossing of DB) is located closely enough for the specified tolerance. Obviously the starting load must be less than the critical load. Also in some cases the initial load increments should not be set too high, as the buckling determinant then could change sign twice within one interval. Judgement will have to be used here, and it might be worthwhile to re-evaluate these after a first trial run.

LL, LB, NOD - these variables must correspond exactly to the number of elements and nodes shown in Table 4.7.1.

CARD 6 (8F10.2)

This card gives the coordinates for one node and is repeated in sequence of the nodes for all nodes. See Figures 4.7.1 to 4.7.5.

Cols.

1-10	ZOR(J)	Z-coordinate of node J in right-handed global coordinate system.
11-20	YOR(J)	Y-coordinate of node J in right-handed global coordinate system.

CARD 7 (3I5,5X,5F10.2)

This card is repeated once for each element in the sequence of the elements and gives the nodes to which it is connected. For beam elements only one node is given.

Cols.

1-5	INP(IL,1)	Node I of element no. IL. See Section 4.7.2.
6-10	INP(IL,2)	Node J of element no. IL. Omit this value for beam elements.
11-15	IET(IL)	Type of element: = 1 Plate element = 2 Beam element

CARD 8 (3I5,5X,5F10.2)

Cols.

1-5	NBCON	Number of nonstandard boundary conditions including total number of beam elements to be entered. Use blank card when all boundary conditions are standard (see Card 9) and no beam elements. When buckling determinant is read in (JPC(4) = 1), NBCON = the number of beam elements only.

CARD 9 (3I5,5X,5F10.2)

The standard boundary condition is free edge, along the unloaded sides or junction between plate elements but on this card others can be specified. Repeat this card NBCON times (previous card). Omit if NBCON is zero.

Cols.

1-5	NODE	Node of plate element for which the nonstandard boundary condition is specified. Or, in the case of beam elements, the node of the beam element.
6-10	IBCOT	= 1 Node is simply support (plate element) = 2 Node is clamped (plate element) = -X where 'X' is the number of the plate element to which the beam element is attached.

CARD 10 (I10,7F10.2)

In the loop on elements the sequence of CARD 10 to 12 is used for plate elements. CARD 13 and 14 are used for beam elements.

Cols.

1-10	L(I)	Number of laminas in plate element No. I.

CARD 11 (9F8.2,4X,I4)

This card gives the thickness and material properties for one layer so therefore provide one Card Type 11 for each layer in the laminate. The layers have to be given in sequence starting from a reference plane, which is located at the surface of the plate element that correspond to the negative z-axis in the local coordinate system.

The properties can be entered in 4 different ways according to the option control in cols. 77-80.

a. Give the engineering constants E11, E22, ν12, G12 for orthotropic laminas when known. For isotropic laminas E22 need not be given.

b. Give fiber and matrix properties and volume fraction coefficient. Continguity factors have no change for this lamina. See Card Type 12.

c. Same as (b) but contiguity factors change, so set control for later read. See Card Type 12. If continguity factors are not given for the first lamina they assume values of zero, until another Card Type 12 is entered.

d. Give the lamina stiffness matrix Q directly.

Cols.

1-8	T	Thickness of lamina (in.).	
9-16	E11,	E11	E-modulus (option 0).
		= EF	E-modulus for fibers (option 1 and 2).
		= Q11	Element of lamina stiffness matrix, Q (option 3).
17-24	E22,	E22	E-modulus for direction 2 (option 0). E22 need not be entered for isotropic laminas. (blank)
		= GF	G-modulus for fibers (option 1 and 2).
		= Q12	Element of lamina stiffness matrix, Q (option 3).
25-32	RNUA,	RNUA	Poisson's ratio 12 (option 0).
		= ZMUF	Poisson's ratio for the fibers (option 1 and 2).
		= Q22	Element of lamina stiffness matrix, Q (option 3).
33-40	G12,	G12	G-modulus (option 0).
		= EM	E-modulus for matrix material (option 1 and 2).
		= Q66	Element of lamina stiffness matrix, Q (option 3).
41-48	GM	Shear modulus for matrix material (option 1 and 2 only).	
49-56	ZMUM	Poisson's ratio for matrix material (option 1 and 2 only).	

Cols.

57-64	VFC	Volumn fraction coefficient of fibers (option 1 and 2 only).
65-72	ANGLE	Angle of the ply in degrees. Only 0° and $\pm 90^\circ$ are permitted for full buckling calculations. Any angle can be used when only material properties are calculated (option 1 and 2 only). See CARD 2, cols. 76-80.
73-76	NOT USED	
77-80	CO	Option control. There are 4 choices:

(opt. 0) CO = 0 Give elastic constants E11, ν 12, G12 for isotropic laminas, for orthotropic laminas give also E22.

(opt. 1) CO = 1 Give the properties of the laminas in terms of fiber and matrix properties. Contiguity factors are not changed for this lamina.

(opt. 2) CO = 2 The same as Option 1, but the contiguity factors changed for this lamina.

(opt. 3) CO = 3 Give the lamina stiffness matrix, Q, directly.

CARD 12 (8F10.2)

This card contains the contiguity factors CONT1 and CONT2 to be used when a layer's properties are given as matrix and fiber properties. This card will follow the material properties-card which has the flag CO set to 2 (Card Type 11). Subsequent layers will use the same contiguity factors unless a change is introduced with another CARD 11 with the appropriate flag CO = 2. If no CARD 12 is used CONT1 and CONT2 will be set to zero and if constant contiguity factors other than zero are required then enter them with the first orthotropic lamina.

Cols.

1-8	CONT1	Contiguity factor to be used for computation of G-modulus and ν12.
9-16	CONT2	Contiguity factor to be used for computation of E22 modulus.

NOTE: The formulae used for calculation of material constants when matrix and fiber properties are known have been taken from:

> Tsai, S. W., "Structural Behavior of Composite Materials," NASA-CR-71, Section 2.0 (1964)

and

> Ashton, J. E., Halpin, J. C., Petit, P. E., "Primer on Composite Materials: Analysis Progress in Material Science Series," Vol. III, Chapter 2.3, Technomic Publications, 1969.

CARD 13 (9F8.2,2I4)

This card gives the dimensions and material properties for a beam element (sequence 10 to 12 is used for plate elements). Three types of beam elements are permitted according to an option control (see cols. 72-80 of this card for description). These types are: general beam element (of any shape), rectangular, and circular. The latter two may also be laminated, with up to 25 laminas, in which case this card gives only the thickness and properties of the first lamina, (inner lamina in the case of circular) the number of laminas and offsets. Card Type 14 gives the information for the other laminas.

Cols.				
1-8	EB			E-modulus of beam element in longitudinal direction.
9-16	GB			G-modulus of beam element material.
17-24	AFB,	as	AFB	Area of beam element (option 0).
		or	TB	Thickness of the first lamina of the rectangular beam (option 1). The thickness is the dimension of the beam element measured parallel to the local y-axis. See Figure 2.10.
		or	RB	Outer radius of the first lamina of the laminated circular beam element (option 2). For a hollow circular beam element the hollow part forms the first lamina, with zero E and G moduli.
25-32	RIYB	as	RIYB	Moment of inertia about yy-axis (option 0).
		or	WB	Width of beam element (option 1).
33-40	RIZB			Moment of inertia about zz-axis (option 0 only).

Cols.

41–48	RGAM	Warping constant for beam element (option 0 only). The program sets the polar moment of inertia RIP equal to RIYB + RIZB.
49–56	RJB	Torsion constant of bead or tip (option 0 only).
57–64	ALFX	Angle between the local y-axis of beam element and the global Y-axis (measured clockwise, from the global Y-axis).
65–72		NOT USED
73–76	NLAM	Number of layers if laminated beam element (option 1 and 2 only) (≤ 35)
77–80	CO2	Option Control. The geometry and material constants of the beam element can be input in three different ways:
	= 0	In this option the program does not distinguish between the type of beam element as all the section properties are calculated and entered by the user. No laminated beam elements are permitted under this option.
	= 1	A beam element of rectangular cross section. Enter the thickness and width and the program will establish the beam element cross section properties.
	= 2	A circular beam element (bead) is used. Enter the radius, and section properties are computed.

CARD 14 (9F8.2,2I4)

This card is used only in connection with laminated beam elements and furnishes the thickness and properties for laminas other than the first one. For laminated circular beam elements, the inner most lamina is the first lamina.

Cols.

1–8	EXX(IR,I)	E-modulus of lamina No. I of beam element No. IR in longitudinal direction.
9–16	BGA(I)	G-modulus for lamina No. I of beam element material.
17–24	TBA(I)	Thickness of lamina No. I of rectangular beam element. In the case of circular beam element this gives the radius to outside of lamina No. I.

CARD 15 (16I5)

Cols.

1-5	ISTOF	Number of elements with nonstandard offsets for plate elements; i.e., elements where a plane different than the midplane is used.

CARD 16 (I5,3F10.2)

Repeat this card for each element affected and omit if ISTOF = 0.

Cols.

1-5	ILN	Element number.
6-15	OFF1	Offset z_o in the local z direction at the starting end $(y=-b/2)$ of the plate element measured positive in the positive z direction, from the negative z surface of the element to the grid. See Note below.
16-25	OFF2	Offset y_o in the local y direction at the starting end $(y=-b/2)$ of the plate element, measured positive in the positive y direction, from the end of the element to the grid. See Note below.
26-35	OFF3	Offset z_o at the end $y=+b/2$ of the plate element. Measured similarly to OFF1.
36-45	OFF4	Offset y_o at the end $y=+b/2$ of the plate element. Measured similarly to OFF2.

NOTE 1:

The grid defines the cross-section such that the plate elements are represented by lines which normally runs along the midplane of each plate element. Nonstandard offsets are read in to define elements where this is not possible, for instance the case of two elements which meet at 180° but are not flush with each other. See Figures 4.7.1 to 4.7.5 and Section 4.7.3.

NOTE 2:

The offsets to be used when the plate element has a beam element adjacent to it are measured in a similar way but they are measured to the geometric center of the beam element.

CARD 17 (16I5)

Dimensions of type matrices for buckling determinant blocks. Omit if JPC(4) = 0.

Cols.			
1-5	IDA1	Number of columns in type matrix ITYPA.	First Block
6-10	IDA2	Number of rows in type matrix ITYPA.	
11-15	IDM1	Number of columns in type matrix ITYPM.	Repetition Blocks or Mid-Block
16-20	IDM2	Number of rows in type matrix ITYPM.	
21-25	IDB1	Number of columns in type matrix ITYPB.	Last Block
26-30	IDB2	Number of rows in type matrix ITYPB.	
31-35	IOLX	Overlap between ITYPA and ITYPM and between ITYPM and ITYPB. (NOTE: The blocks must be structured such that their overlaps are equal.)	

CARD 18 (16I5)

Repeat for each successive row of ITYPA-ITYPM, and ITYPB matrices. Omit if JPC(4) = 0.

Cols.	
1-5	ITYPA(1, 1)
6-10	ITYPA(1, 2)
11-15	Etc.

Use similar card type for ITYPM and ITYPB.

4.7.1 Data Card Sequence

For a typical problem the cards are stacked in the following sequence:

	Card type	
Title Card	1	
Controls, intermediate print, etc.	2	
Panel type, number of blocks, displace. opt. cont., type matrix control	3	
Axial mode, limits, options	4	
No. of plate elements, beam elements, nodes, length, startload, load intervals	5	
Coordinates - one card per node	6	
Element data - one card per element	7	
Number of nonstandard boundary conditions	8	
Boundary conditions	9	

Number of laminas	10	A, C
Thickness, material properties, option for plate element	11	A, C
Contiguity factors	12	A, C

Repeat sequence A as required by number of laminas in current plate element and change in contiguity factors.

Use sequence C for plate elements in the section.

Beam element (if any) properties, geometry, and option	13	B
Beam element properties for laminas other than the first if beam element is laminated	14	B

Repeat Card 14 as required by number of laminas. Sequence B is used only for beam elements.

Number of nonstandard offsets	15
Offsets	16
Dimensions of type matrices to be read	17
Type matrices ITYPA, ITYPM, ITYPB	18

NOTE:

The lamina properties cards for the plate element must be stacked sequentially in the direction of the positive z-axis of the local coordinate system for the element. The orientation of local coordinate systems are demonstrated in Figure 4.7.1 to 4.7.5.

TABLE 4.7.1

STANDARD TYPES OF PANELS

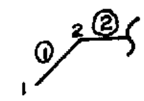
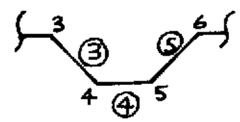
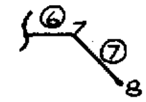
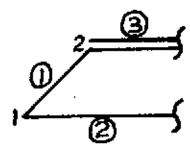
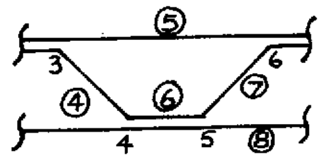
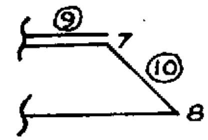
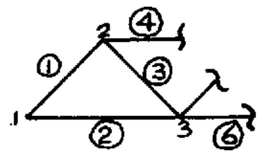
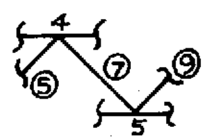
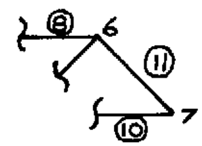
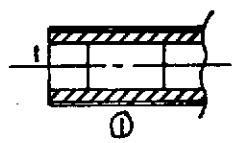
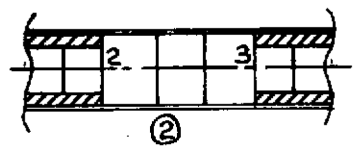
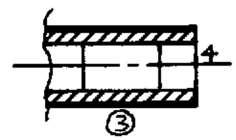
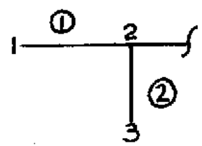
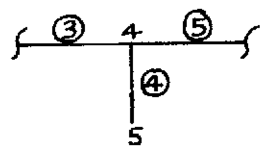
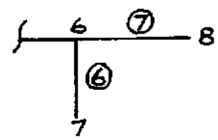
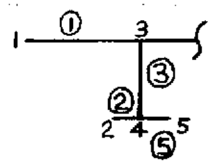
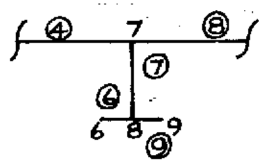
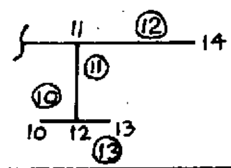
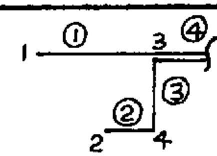
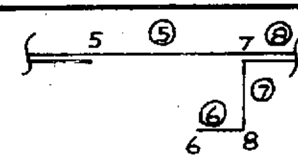
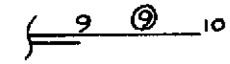
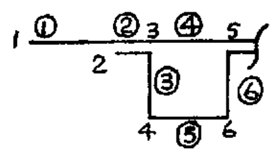
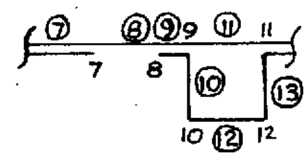
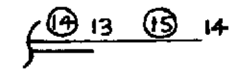
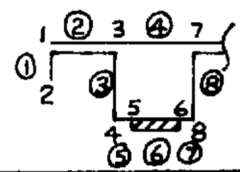
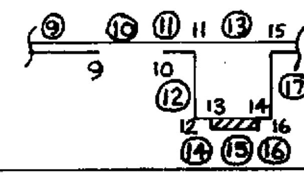
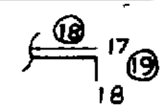
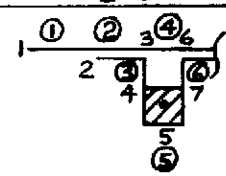
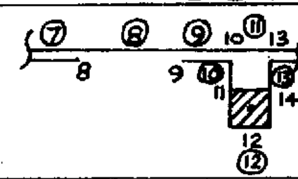
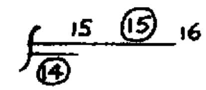
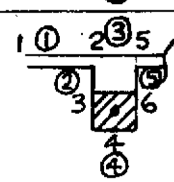
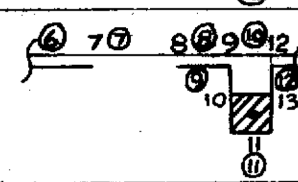
Type No.	Description	First Block	Repetitive Block	Last Block
10	Corrugated Plate			
20	Corrugated Core Sandwich			
30	Truss Cord Sandwich			
40	Honeycomb Core Sandwich with reinforcements			
50	Integral Panel			
60	Integral Panel with Tee Stiffeners			

TABLE 4.7.1 - continued

STANDARD TYPES OF PANELS

Type No.	Description	First Block	Repetitive Block	Last Block
70	Zee Stiffener Panel			
80	Hat Stiffener Panel			
81	Hat Stiffener Panel with Local Reinforcement			
90	Angle Stiffener Panel with local Reinforcement			
91	Angle Stiffener Panel with local Reinforcement			

4.7.2 Node and Element Numbering System

The table of standard types of panels shows the node numbering system. Each flat plate element has two nodes (one at each end) and the beam element has one node (at the geometric center).

A right handed global Y-Z axes is chosen. (Z Y). Y axis is made to coincide with the midplane of one or more of the plate elements. The nodes are numbered in the increasing Y direction, starting from the left side of the panel. If there are two nodes at the same Y coordinate (see for example nodes 2 and 3 of the Integral Panel, Type 50) the nodes at that Y location are numbered in the increasing Z direction. The table of standard types of panels illustrates the node numbering system for various panels where the global axes are omitted for clarity. The node numbers are shown uncircled. The nodal coordinates (Y and Z) with respect to the chosen global axes are now easily fixed.

Elements are numbered in sequence, proceeding in the increasing order node numbers, starting with node 1. The element numbers are shown circled in the table of standard types. Panel types 90 and 91 illustrate the numbering system when beam elements are involved.

Any other panel can be numbered in a similar way.

4.7.3 Local Coordinates and Off-Sets y_o and z_o

Local y axis is chosen in the increasing direction of node numbers of the ends of the plate element. See Figure 4.7.1.

The local y-z axes form a right handed system.

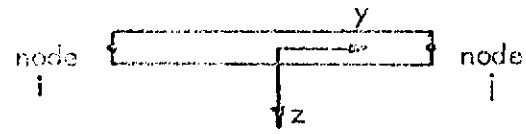


FIGURE 4.7.1

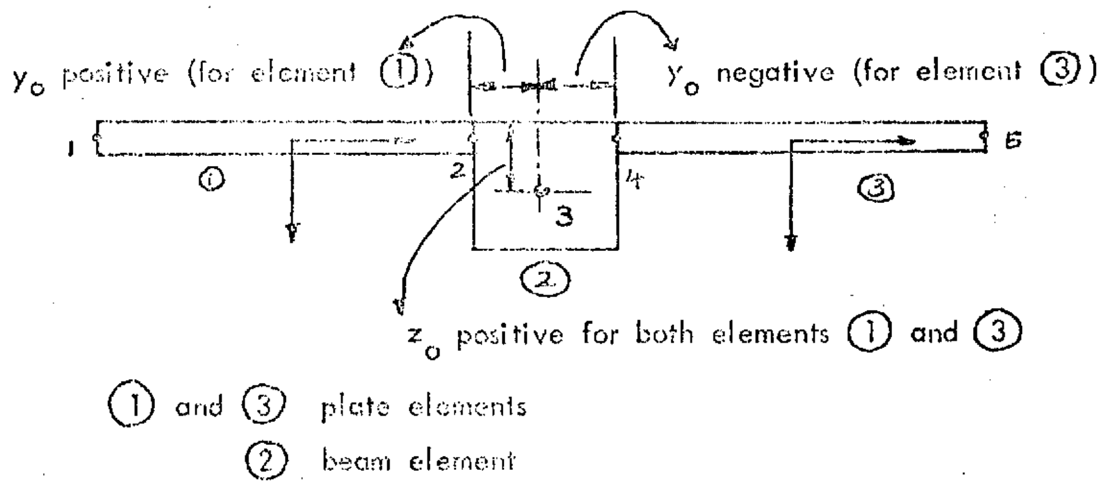


FIGURE 4.7.2

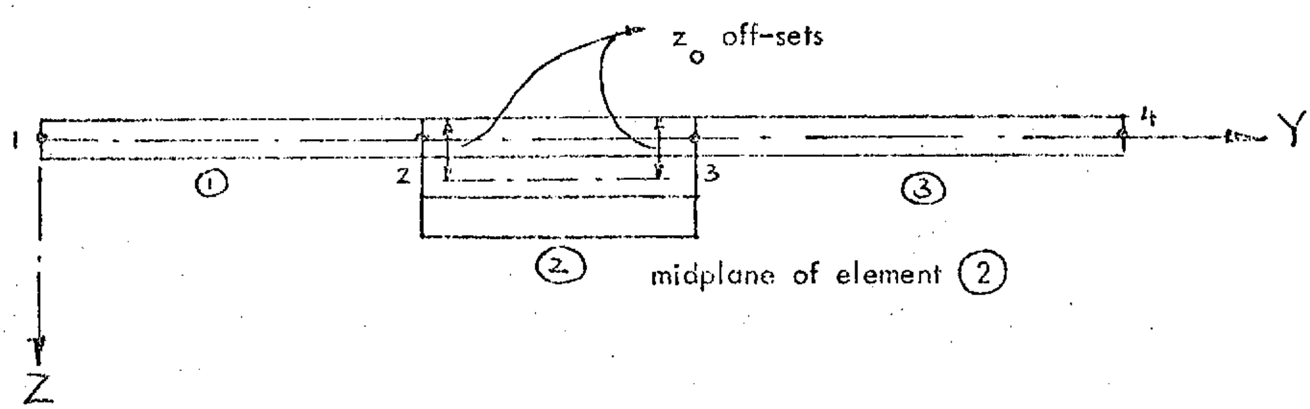


FIGURE 4.7.3

The off-set z_o is measured positive from the negative z surface of the element, in the positive z direction. The off-set y_o is measured positive from the end $y = + b/2$ or $y = - b/2$, in the positive y direction.

These off-sets are used when the nodes are not in the mid-plane of the element. (Note: Nodes are fixed with respect to global axes.)

When there are beam elements involved, the off-sets are given for the adjacent plate elements. (See Figure 4.7.2)

The off-sets when plate elements of different thicknesses meet are illustrated in Fig. 4.7.3.

The z_o off-sets can either be used on element 2 when the global Y axis is as shown or on element 1 ($y = + b/2$) and on element 3 ($y = - b/2$) when the global Y-axis coincides with midplane of element 2.

4.7.4 Buckling Determinant Input (JPC(4) = 1 in Card 3)

Any panel or structural section, of type not covered by the standard type numbers (Table 4.7.1) can be run on the program by reading in the "buckling determinant." The general principles of forming the "buckling determinant" are given in Sections 2.8 and 2.9. However, a more detailed description is given in this section together with typical examples.

After idealizing the structure into flat plate elements and beam elements, the nodal numbering and the element numbering are done as per Section 4.7.2. The local coordinate system and the off-sets are fixed as per Section 4.7.3.

Basically, each column of the "buckling determinant" represents a single element of the structure. The columns are arranged in the order of increasing element numbers. Each row of the determinant represents either the displacement continuity between two elements at a time, of all the elements at a junction of elements until the displacements of all elements are equated, or the force equilibrium between all the elements at the junction. Thus 'n' elements at a junction yield (n-1) rows from the displacement continuity and one row from the force equilibrium considerations. Each flat plate element at a junction can be individually free (zero forces), simply supported or clamped. Such condition yields one separate row in the "buckling determinant," for each such flat plate element, at the junction. It is pointed out that the minimum number of elements possible at a junction is one flat plate element, in which case it is not interconnected to any other element. Also, a beam element is always connected to at least one flat plate element.

The submatrix type numbers given below are used in forming the buckling determinant (see also Table 2.4).

Type No.	Description
1	Displacements at the node I ($y = - b/2$) of the flat plate element. (Note: The displacements as given by Type 1 have positive directions opposite to that of Type 3.)
2	Forces at the node I ($y = - b/2$) of the flat plate element.
3	Displacements at the node J ($y = + b/2$) of the flat plate element.
4	Forces at the node J ($y = + b/2$) of the flat plate element.

Type No.	Description
7	Displacements of a beam element attached to the node I ($y = -b/2$) of the flat plate element. (Note: The displacements as given by Type 7 have positive directions, same as Type 3.)
8	Forces of a beam element attached to the node I ($y = -b/2$) of the flat plate element.
9	Displacements of a beam element attached to the node J ($y = +b/2$) of the flat plate element. (Note: The displacements as given by Type 9 have positive directions opposite to that of Type 3.)
10	Forces of a beam element attached to the node J ($y = +b/2$) of the flat plate element.
11	Simply supported boundary conditions at node I ($y = -b/2$) of the flat plate element.
12	Clamped boundary conditions at the node I ($y = -b/2$) of the flat plate element.
13	Simply supported boundary conditions at the node J ($y = +b/2$) of the flat plate element.
14	Clamped boundary conditions at the node J ($y = +b/2$) of the flat plate element.

NOTE: Submatrix Types 2 and 4 also represent the boundary conditions of a free edge (zero forces) at nodes I ($y = -b/2$) and J ($y = +b/2$), respectively, of a flat plate element.

The element junctions are considered one at a time. For those elements at a junction which are individually free, simply supported or clamped, the corresponding submatrix type number is entered in the column corresponding to the element number in the "buckling determinant," each element contributing to a separate row. The interconnected elements at the junction are then considered two at a time. The appropriate displacement submatrix type numbers (1, 3, 7, or 9) are each time entered in one separate row and in columns corresponding to these two element numbers involved. When two displacement submatrices of identical type number occur in the same row, one of them is given a negative sign, e.g.

$\begin{array}{|c|c|}\hline 3 & -3 \\ \hline\end{array}$ or $\begin{array}{|c|c|}\hline 1 & -1 \\ \hline\end{array}$

After one has thus equated the displacements of all the interconnected elements at a junction taking two elements at a time, the next row of the "buckling determinant" is formed from the appropriate force submatrix type numbers (2, 4, 8, or 10) in the columns corresponding to all the interconnected elements at the junction.

The rest of the "buckling determinant" is then completed in a similar fashion until all the junctions are covered.

In most cases it is more efficient to divide the buckling determinant into three blocks - a start block ITYPA, a mid block ITYPM (this can also be the repetitive block representing the repetitive nature of part of the structure as in a stiffened panel) and end block ITYPB. For these cases the overlap between the blocks must be the same in terms of the number of submatrix types. If possible, it is also suggested to make ITYPA bigger than ITYPM or ITYPB in order to reduce the storage required by the determinant evaluator subroutine BLKDET.

Three examples are given below to show the "buckling determinant" formed as described above.

EXAMPLE 1: All Plate Elements, Figure 4.7.4.

Buckling Determinant:

11		
3	1	
3		1
4	2	2
	4	
		13

or

11		
3	1	
	1	-1
4	2	2
	4	
		13

The three columns correspond to the 3 elements. The first junction considered (node 1) has the flat plate element No. 1 only there. The side y = - b/2 of the element is simply supported. Hence the type No. 11 in the first row of the first column. The next junction considered is that at node 2. Flat plate elements 1, 2, and 3 meet at this junction. Two elements are considered at a time. Considering 1 and 2 first and their appropriate displacement submatrices, yield type 3 in column 1 and type 1 in column 2 of the second row. Next one can either consider elements 1 and 3 or elements 2 and 3 as the next pair. This yields type 3 in column 1 and type 1 in column 3 or type 1 in column 2 and type (-1) in column 3 of the third row. The displacements of all the elements are thus equated taking two of them at a time. The next row of the "buckling determinant" is formed from the force submatrix types of all the elements at the junction under consideration (node 2). This yields the fourth row with types 4, 2 and 2 in columns 1, 2 and 3, respectively. The next junction which is node 3 has the flat plate element No. 2 with a free edge (y = + b/2). This yields type 4 in column 2 of row 5. The last junction which is node 4 and has the flat plate element No. 3 with simply supported edge (y = + b/2) yielding type 13 in column 3 of the last row.

The "buckling determinant" can be split into 3 blocks, the overlap between the blocks being the same, as below.

Start Block:

11

(1 x 1)

Mid Block:

3	1	
3		1
4	2	2
	4	

or

3	1	
	1	-1
4	2	2
	4	

(4 x 3)

Last Block:

13

(1 x 1)

Overlap:

1

EXAMPLE 2: Two Plate Elements (1 and 3) and One Beam Element (2). (Fig. 4.7.5)

<u>Buckling Determinant:</u>

12		
3	9	
3		1
4	10	2
		14

or

12		
3	9	
	7	1
4	10	2
		14

The three columns correspond to the three elements. The first junction considered, i.e. node 1 yields type 12 in column 1 of the first row. The next junction considered is at node 3 and is between the flat plate element 1, the beam element 2 and the flat plate element 3.

The second, third, and fourth rows of the "buckling determinant" are formed by considerations identical to the junction at node 2 of Example 1, the only difference being that the element No. 2 is now a beam element. The last junction to be considered is that at node 4 which yields the type 14 in the last column of the last row.

The "buckling determinant" is finally split into 3 blocks, as below:

Start Block:

$\boxed{12}$ (1 x 1)

Mid Block:

3	9	
3		1
4	10	2

or

3	9	
	7	1
4	10	2

(3 x 3)

Last Block:

$\boxed{14}$ (1 x 1)

Overlap:

1

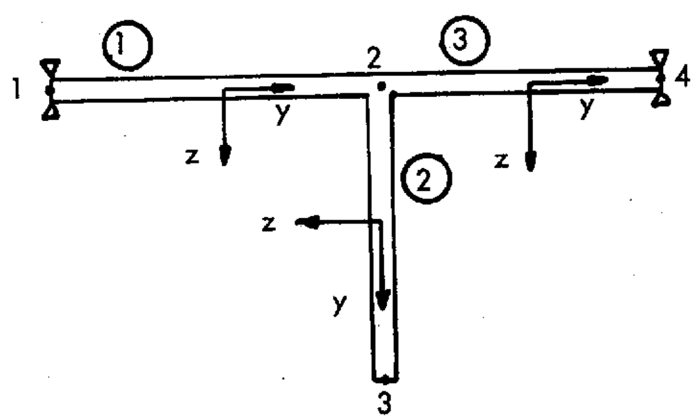


FIGURE 4.7.4

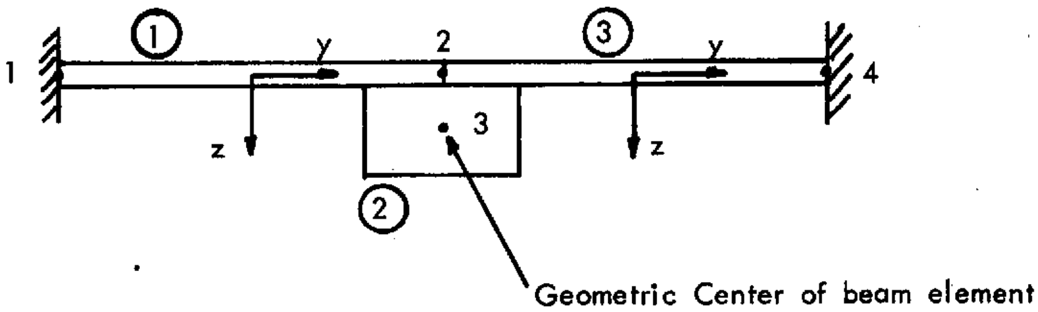


FIGURE 4.7.5

EXAMPLE 3: Stiffened Panel

Figure 4.7.6 shows a typical stiffened panel with stiffeners S_1 and S_2 repeated over the width of the panel. For the purpose of input to the program, the buckling determinant is considered to consist of three parts, namely, a start block ITYPA, a repetitive block ITYPM, and an end block ITYPB. It is pointed out that all repetitive blocks ITYPM have elements identical in all respects. This is illustrated in Figure 4.7.6 where two different ways of forming the blocks are shown, namely, (a) the one shown in the upper half of the figure and (b) the one shown in the lower half of the figure. The upper one applies when two outer plate bays have the same width as the plate between adjacent stiffeners S_1 and S_2. The lower one applies when the two outer plate bays have widths differing from the plate width between adjacent stiffeners S_1 and S_2.

In forming the "buckling determinant" a "reduced panel" consisting of the start block ITYPA, one repetitive block ITYPM and the end block ITYPB, as shown in Figure 4.7.7 is considered. This corresponds to the blocks shown in the upper half of Figure 4.7.6. The node numbers and the element numbers are also shown in Figure 4.7.7.

There are 11 elements in the "reduced panel." Thus the "buckling determinant" has 11 columns.

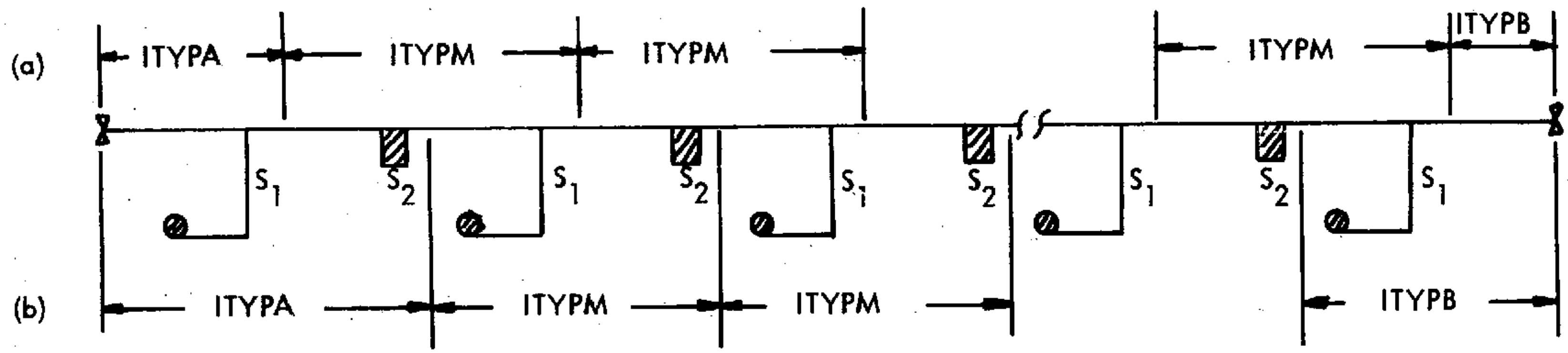


FIGURE 4.7.6. Stiffened Panel with Repetitive Stiffeners

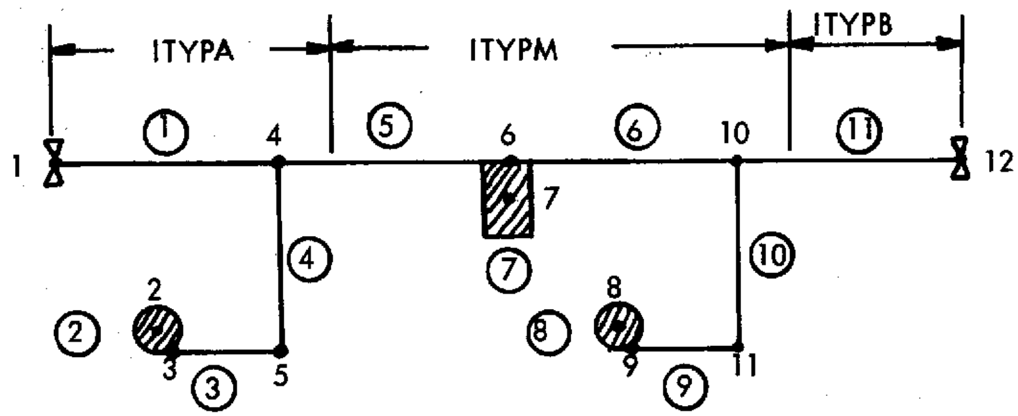


FIGURE 4.7.7. "Reduced Panel" for Program Input

(Note: Circled numbers are element numbers)

11				
	7	1		
	8	2		
3			1	
3				1
4			2	2
		3	-3	
		4	4	

ITYPA

3	1					
3		9				
4	2	10				
			7	1		
			8	2		
	3				1	
	3					1
	4				2	2
				3	-3	
				4	4	
						13

ITYPM

ITYPB

The element junctions are now considered one by one. The first junction which corresponds to node 1 has the flat plate element (1) there simply supported along $y = -b/2$. Hence the type No. 11 is entered in the first column of the first row. The next junction considered is that corresponding to node 3. The elements at this junction are the beam element (2) and the flat plate element (3). The displacement submatrix types 7 and 1 are thus entered in the second row in columns corresponding to these element numbers. Since there are only two elements at this junction, the force submatrix types 8 and 2 are in a similar manner entered in the next row.

The junction considered next corresponds to node 4, and has three flat plate elements (1), (4) and (5). Two elements are considered at a time, till the displacements of all the elements are equated. Thus the appropriate displacement submatrices of elements (1) and (4) are entered in the third row and those for elements (1) and (5) in the fourth row, in columns corresponding to the element numbers. The choice of element pair (1) and (5) is arbitrary. Instead one could choose for the fourth row, the element pair (4) and (5), with the appropriate submatrix type numbers in the right columns. Having thus equated the displacements of all elements at the junction being considered, the next row is formed from the appropriate force submatrix types of all elements at the junction, i.e. (1), (4), and (5), in their right columns. The rows 7 and 8 are formed in similar manner considering the junction at node 5. The junction considered next is at node 7 and has the flat plate elements (5) and (6) and the beam element (7). The rows 9, 10, and 11 are formed following the same procedure as with the other junctions. In equating the displacements two by two, the element pair (5) and (7) are chosen arbitrarily, in row 10. In this choice the beam element (7) is attached to the side $y = + b/2$ of the flat plate element (5). Hence the type numbers used for the beam element (7). The rest of the "buckling determinant" is formed in a similar manner. It is easy to see what forms ITYPA, ITYPM, and ITYPB. The overlap is alos evident.

4.8 Output From Program

The output from this program is given only in the form of printed output. First, the input data is printed out and labeled for easier checking. See Section 4.7 for identification of input data and Section 6.0 for sample problem.

4.8.1 Geometry

The control information regarding type of section, beam element, and boundary conditions of the section is interpreted and messages printed. The first variable which is computed and printed out is z_n for each plate element. z_n is the distance from a reference plane at one surface to an established neutral plane and thus describes its location. The lamina stiffness matrix Q for each lamina is then printed and after this comes the A-, B-, and D-matrices for each plate element. The A-matrix represents the extensional stiffness of the plate element, while the B- and D-matrices are the coupling stiffness and bending stiffness, respectively. Also the angles between the plate elements and the actual widths of the plate elements are printed. Three matrices ITYPA, ITYPM, and ITYPB that identify submatrices of the buckling determinant are set up and printed.

4.8.2 Buckling Loads

The next phase is the actual buckling calculations. The loop on the specified number of modes prints out first a label for identification of the mode and then the AB, AB2, RES, and ICOM arrays.

The AB-array contains the line loads on plate element No. 1 for each trial load in the search for a critical load while AB2 is used for the total load on the section. RES contains the respective values of the buckling determinant. ICOM contains appropriate comments describing the action that was taken at the time by the program.

During the search for a critical load, some loads may give double roots in the solution of the equilibrium equations. Since the signs of the buckling determinant to either side of the load which cause the double root are not relevant to each other, this double root has to be zeroed in. The appropriate messages are printed when this occurs.

After the loops on the modes are completed the buckling loads for all the modes are printed out. The mode with the lowest buckling load is picked as the critical one and printed out with its mode. For reasons of identification the title of the run is printed in strategic places. Timing information is printed for each mode and for the total data set.

4.8.3 Eigenvector Output

The overlay for the eigenvector and the relative displacements (3.0) prints first identification of the data set as given in the title card.

Then the bandwidths are printed. The core requirements for the eigenvector solution including the dynamic storage allocation in blank common is shown, for the current problem together with how many stiffeners can be run for a field length of 70K.

After the buckling determinant has been transformed to a compact banded form and the decomposition is done the buckling determinant value is printed for checking purposes.

Each iteration of the eigenvector is printed together with the used normalizing factor.

The routine DIS which establishes the relative displacements first prints out the roots of the equilibrium equations. Thereafter the relative displacements u and v at certain points across the width of each plate element are printed out together with the local coordinate y. For beam elements u and v are given for the geometric center of the cross-section of the beam element.

5.0 VERIFICATION OF RESULTS

Two types of checks are made to verify the program; namely,

(a) Engineering tests, to correlate the results from the program with results available in literature (Section 5.1).

(b) Functional tests, to check all major program logic, that is not covered by (a) (Section 5.2).

5.1 Engineering Tests

As stated earlier, the purpose of the "engineering tests" are to correlate the results from the program with any results available in literature. For convenience, these tests are separated into the following groups:

(a) Test problems specified by NASA.

(b) Additional test problem from literature.

5.1.1 Test Problems Specified by NASA

The outline of these test problems as given by NASA are given in Appendix B.

TEST PROBLEM 1:

Table 5.1 shows the geometric and material data of a simply supported web with an orthotropic flange. The basic section is of aluminum alloy and the flange is symmetrically reinforced with boron fiber composite (0°). The geometry is so chosen that buckling corresponds to the "local buckling" mode defined in Reference 1 of Appendix B. The flange and the web are idealized as flat plate elements, as in the reference literature. Table 5.1 shows the results from the program. Figure 5.1 shows the same results superposed on the results from the reference literature.

For the "beam-column" mode of Problem 1 of Appendix B, the following geometry is used:

b_w = 1.5 ins.; b_F = 0.6 ins.

t_w = 0.05 ins.; t_F = 0.15 ins.

Length a = 15.0 ins.

The material properties used are the same as in Table 5.1. The orthotropic flange is idealized as a beam element and web as a flat plate element, as in Reference 1 of Appendix B. At buckling, the load per unit width of the web, $(\bar{N}_{11})_w$, from the program is 835.3 lb/in. This gives a value of 1.58 for

$$k_w = \left(\frac{(\bar{N}_{11})_w b_w^2}{\pi^2 D_w} \right) ,$$

as against the value of 1.56 quoted by NASA in Appendix B.

Figure 5.2 shows the plot of the buckling mode shape results from the program for the "local buckling" mode ($\lambda/b_w = 2.5$) and the "beam column" mode.

TEST PROBLEM 2:

Figure 5.3 shows the geometry and the material properties of two 60° truss core sandwich plates. Figure 5.4 shows the results from the program, together with the results from Reference 2 of Appendix B. Also shown are the plots of the buckling mode shape results from the program.

TEST PROBLEM 3:

Figure 5.5 gives the geometric data, material data and the results from the program for two simply supported aluminum alloy plates with a single eccentric boron fiber (0°) composite deep stiffener. The results from the program are also shown superposed on the results from Reference 1 of Appendix B. The plate and the stiffeners are idealized as flat plate elements as in the reference literature.

Table 5.2 gives the geometric and material data for a series of simply supported aluminum alloy plates with a single eccentric boron fiber (θ°) composite shallow stiffener. The results from the program are tabulated in Table 5.3. It is seen that two sets of values (I and II) are quoted as results from the program. In both cases the stiffeners are idealized as beams with $G_{23} = 0$, as in Reference 1 of Appendix B.

However, for the results I, the boundary conditions along the unloaded edges of the plate were $w = M_{22} = N_{12} = N_{22} = 0$, as in the reference literature. Since this combination of boundary conditions is not available in the program, it was altered for one run only, to check this particular test problem. In Figure 5.6, the results I from the program are shown superposed on the results from Reference 1 of Appendix B.

The boundary conditions for the unloaded edges of any simply supported plate, as in the program are $w = M_{22} = u = N_{22} = 0$ (i.e., u replacing N_{12} in the previously quoted boundary conditions). The results II in Table 5.3 correspond to these boundary conditions.

Typical buckling mode shape results from the program for simply supported plates with single eccentric deep and shallow stiffeners, respectively, are shown in Figure 5.7.

TEST PROBLEM 4:

Figure 5.8 shows the cross-sectional geometry and the material data of T-section and integrally stiffened plates, with six stiffeners on each plate.

The lengths of the plates are so chosen that the buckling mode corresponds to the "local buckling" mode defined in Reference 4 of Appendix B. Thus, Plate A has a length of 15.0 inches and Plate B a length of 12.3 inches. These stiffened plates are idealized to consist of flat plate elements only, as in the reference literature. The results from the program including the buckling mode shape are shown in Figure 5.9, together with the results from Reference 4 of Appendix B.

For "general instability" as defined in Reference 5 of Appendix B, the length of integrally stiffened plate, Plate B, is increased to 25.0 inches.

Two results are shown in Figure 5.10 for "general instability" of Plate B, one where the integral stiffeners are idealized as beam elements as in the reference literature and the other where the stiffeners are idealized as flat plate elements. Results from the reference literature are also quoted. Buckling mode shapes for these two cases are also shown in Figure 5.10.

The correlation for the above discussed NASA specified problems are seen to be good.

5.1.2 Additional Test Problems

The results for a few other test problems, in addition to the NASA test problems discussed in Section 5.1.1, are given in this section. For easy reference, the numbering sequence of the test problems are continued from the last section.

The stiffened plates considered are:

(i) Test Problem 5 - Corrugated Core Sandwich Plate
(ii) Test Problem 6 - Integral Zee Stiffened Plates
(iii) Test Problem 7 - Bonded Zee Stiffened Plate
(iv) Test Problem 8 - Hat Stiffened Plate

Figure 5.11 shows the geometry and the material data for these panels. The results from the program and those from literature are shown in Table 5.4. The correlation is seen to be good for Test Problems 5 and 6. The discrepancies in the case of Test Problems 7 and 8 are attributed to the differences between the literature and the present theory in the idealization of the flat plate elements consisting of the attached flange and the skin to which it is attached. In the literature quoted their individual stiffnesses are added whereas in the present theory they are treated as a laminated plate and the overall stiffnesses evaluated, leading to much higher stiffness values. The idealization of the literature quoted permits relative sliding of the attached flange and the skin, which perhaps is closer to a rivetted connection. The idealization of the present theory does not allow such relative sliding and assumes perfect bonding between the attached flange and the skin. This difference in idealization causes the buckling stress and the axial half-wave numbers obtained from the program to be higher.

A similar effect has been reported in: Pride, Richard A.; Royster, Dick M.; Gardner, James E.: "Influence of Various Fabrication Methods on the Compressive Strength of Titanium Skin-Stringer Panels," TN D-5389, NASA, August, 1969. In the test results quoted in this report, the buckling stress of a bonded zee stiffened plate is found to be about 19% higher than a rivetted zee stiffened plate, which is very close to the discrepancy in the results for Test Problem 7, in Table 5.4.

The buckling mode shape results from the program for Test Problems 5, 6, and 8 are shown in Figure 5.12.

The correlation for the test problems discussed in Sections 5.1.1 and 5.1.2 are seen to be good and demonstrate the engineering accuracy of the program.

TABLE 5.1

Simply Supported Web with Orthortopic Flange

(Test Problem 1)

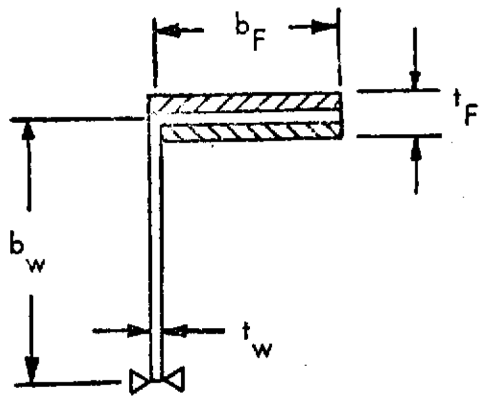
Geometric Data:

$b_w = 2.0$ ins.

$b_F = 0.8$ ins.

$t_w = 0.04$ ins.

$t_F = 3 \times t_w$



Material Data:

Aluminum Alloy:

$E_{11} = E_{22} = 10.5 \times 10^6$ psi

$G_{12} = 4.03 \times 10^6$ psi

$\nu_{12} = 0.3$

Boron Composite:

$E_{11} = 30.25 \times 10^6$ psi

$E_{22} = 2.03 \times 10^6$ psi

$G_{23} = G_{12} = 0.5249 \times 10^6$ psi

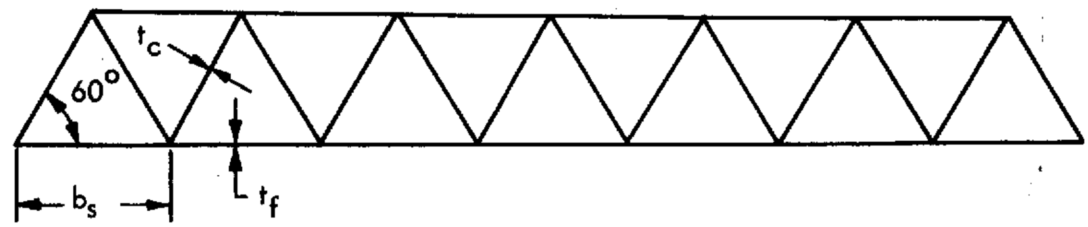
$\nu_{12} = 0.346$

Results from the Program:

λ/b_w	3.0	2.5	2.0	1.33	1.0	.80	.666	.57
$(\bar{N}_{11})_w$ (on web)	665	657	715	902	822	795	833	915
k_w^*	4.41	4.35	4.73	5.97	5.43	5.26	5.51	6.05

$$* \; k_w = \frac{(\bar{N}_{11})_w \, b_w^2}{\pi^2 D_w}$$

λ = Axial half-wave length of buckle $(\frac{a}{m})$.



<u>Geometric Data:</u>

		Plate A	Plate B
Length	a (ins.)	6.0	6.0
	b_s (ins.)	1.0	1.0
	t_f (ins.)	0.02	0.02
	t_c (ins.)	0.02	0.01
Total No. of cells		13	13

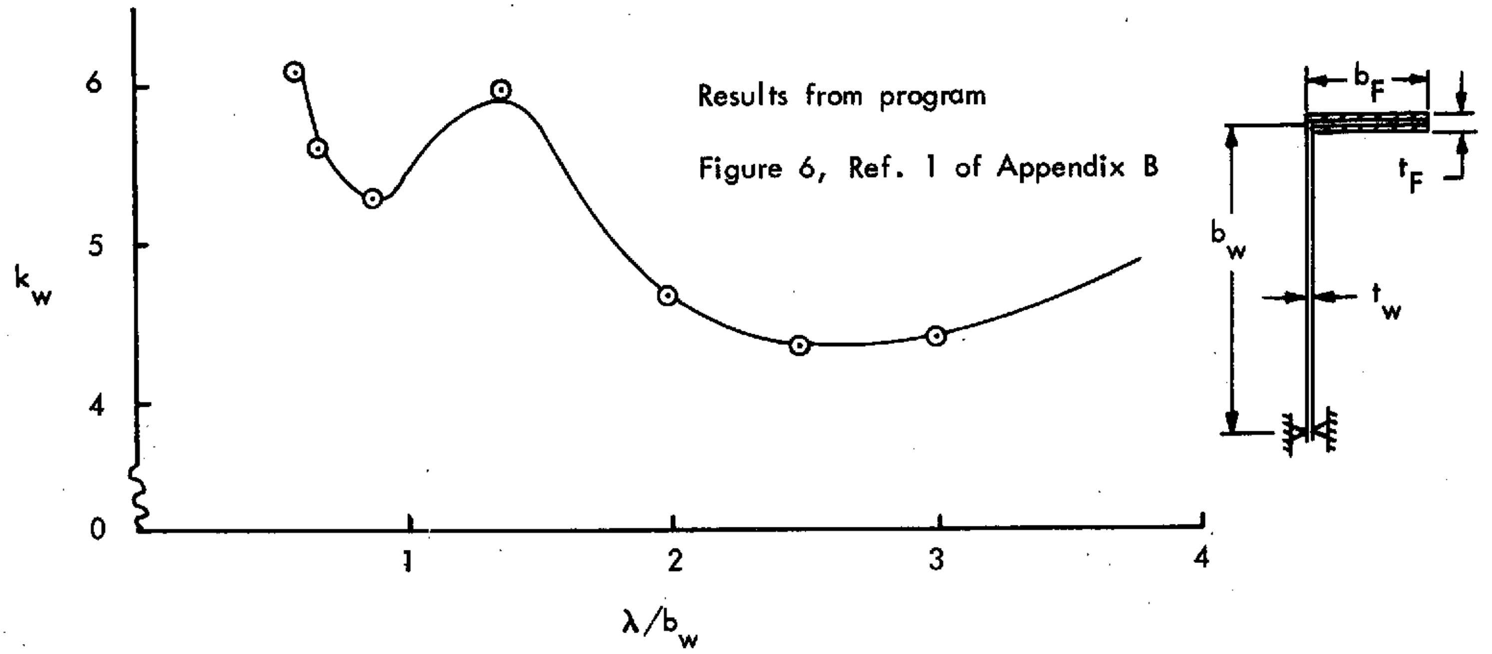
<u>Material Data:</u>

$E_{11} = E_{22} = 10 \times 10^6$ psi

$G_{12} = 3.85 \times 10^6$

$\nu_{12} = 0.3$

FIGURE 5.3. <u>Truss Core Sandwich Plates Data</u>
(Test Problem 2)



λ = Axial half wave length of buckle.

$$k_w = \frac{(\overline{N}_{11})_w b_w^2}{\pi^2 D_w}$$

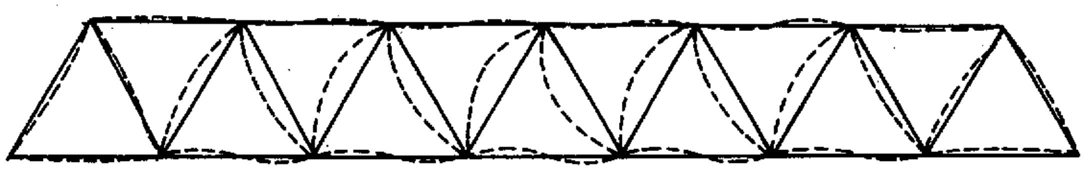
FIGURE 5.1. Simply Supported Web with Orthotropic Flange ($\frac{b_F}{b_w} = 0.40$)
(Test Problem 1)

(a) "Local buckling" mode
($\lambda/b_w = 2.5$)

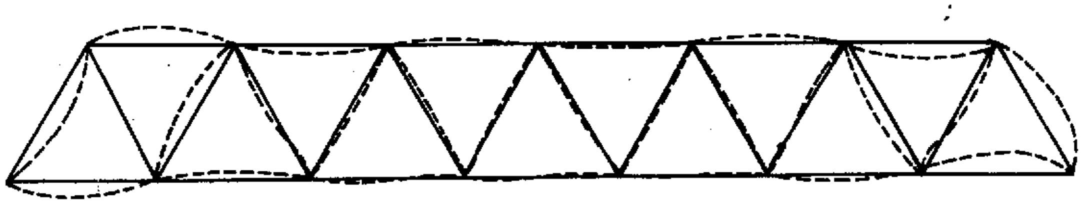
(b) "Beam-column" mode

FIGURE 5.2. Simply Supported Web with Orthotropic Flange - Buckling Mode Shapes
(Test Problem 1)

Plate No.	Results from Program		Critical Stress from Reference 2 of Appendix B (psi)
	Critical Stress (psi)	Axial Half-Wave No.	
A	16954	7	16920
B	6019	9	6070



Buckling Mode Shape for Plate B - Face Restrains Core



Buckling Mode Shape for Plate A - Core Restrains Face

FIGURE 5.4. Results for Truss Core Sandwich Plates

(Test Problem 2)

Geometric Data:

No.	d (ins.)	t (ins.)	m	n	Length a (ins.)
A	3.0	.048	25.0	2.0	30
B	4.0	.032	75.0	2.0	40

Material Data: Same as in Table 5.1.

Results from Program:

No.	N_p (lb/in.)	$\frac{N_p d^2}{\pi^2 D_p}$	No. of half-waves
A	314.9	2.70	5
B	27.4	1.42	4

N_p = critical load (lb/in.) on the plate

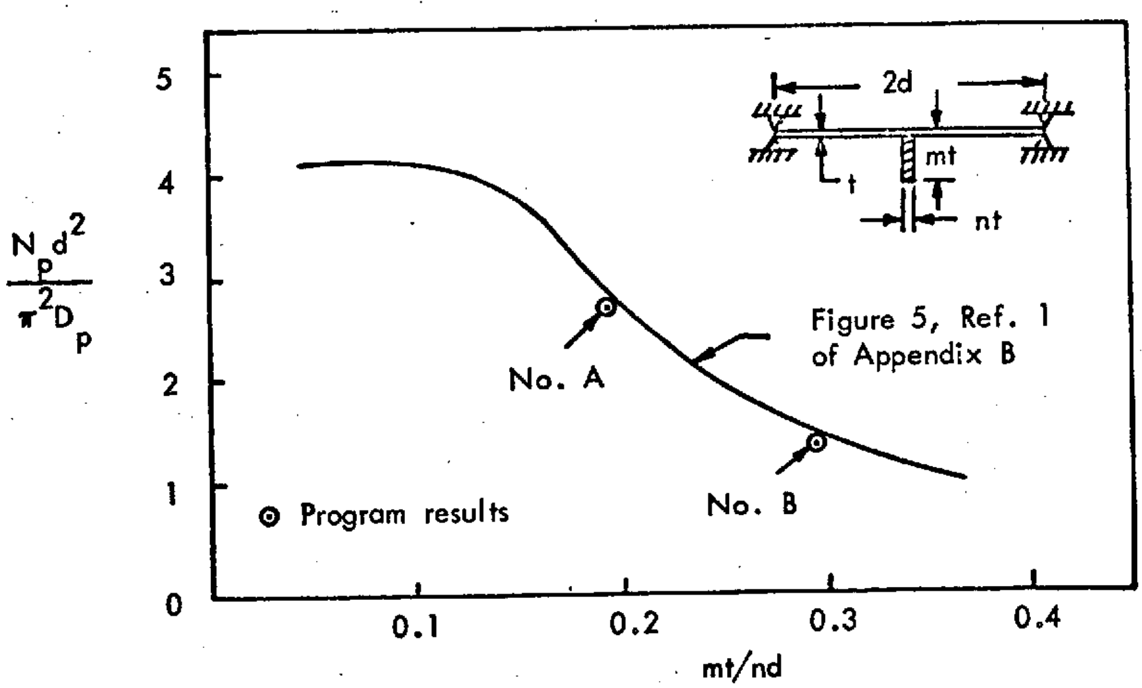
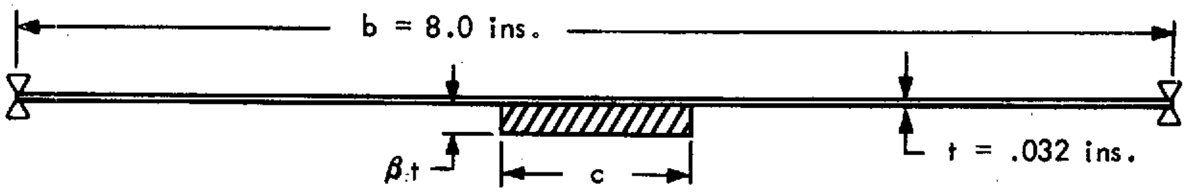


FIGURE 5.5. Simply Supported Plate with Single Eccentric Deep Stiffener
(Test Problem 3)

TABLE 5.2

Simply Supported Plate with Single Eccentric Shallow Stiffener - Data

(Test Problem 3)



a (length of the plate) = 240.0 ins.

Plate No.	β	C (ins.)
C	2.0	2.0
D	4.0	1.0
E	6.0	0.6667
F	8.0	0.5
G	4.0	0.6667
H	4.0	0.334
I	4.0	0.1668
J	4.0	0.0833
K	4.0	0.0

Material Properties:

Same as in Table 5.1, except $G_{23} = 0$ for the stiffener.

TABLE 5.3

Simply Supported Plate with Single Eccentric Shallow Stiffener - Results

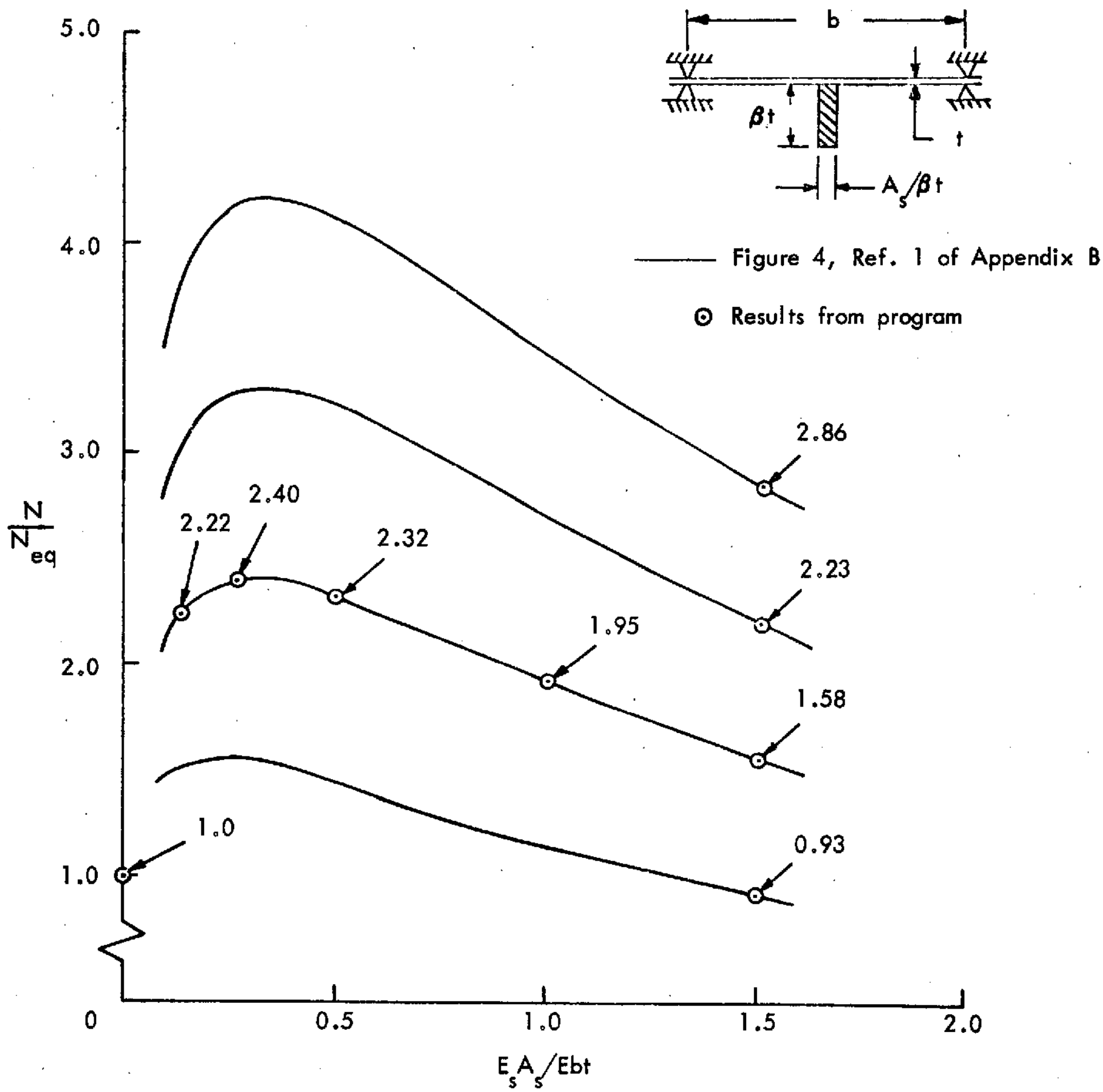
(Test Problem 3)

Plate No.	Results (I) from Program (see note below)		Results (II) from Program (see note below)	
	Critical Load P (lbs)	Axial half-wave no. m	Critical Load P (lbs)	Axial half-wave no. m
C	337.1	13	367.2	13
D	571.0	9	662.2	9
E	806.8	8	972.1	7
F	1034.8	7	1283.6	6
G	542.4	10	608.7	10
H	495.0	11	527.9	11
I	436.6	13	450.1	13
J	370.6	15	375.1	15
K	155.7	30	155.5	30

NOTE:

(1) Results (I) correspond to the boundary conditions $w = M_{22} = N_{12} = N_{22} = 0$ along the unloaded edges. These boundary conditions were specially included in the program for this particular test problem only.

(2) Results (II) correspond to the boundary conditions $w = M_{22} = u = N_{22} = 0$ along the unloaded edges. These boundary conditions are used in the program for any simply supported edge.



E_s, E = Young's Modulii of stiffener and plate, respectively, (psi)

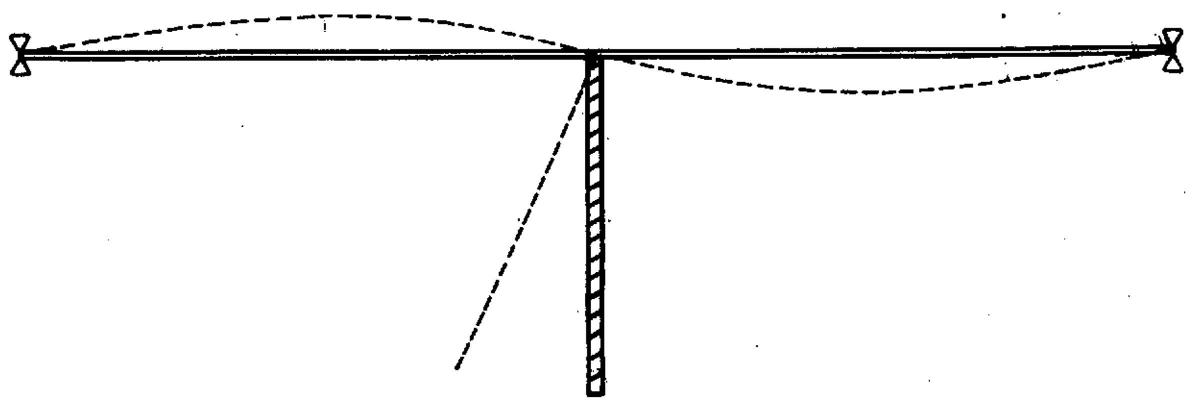
A_s = Stiffener area (in.^{2})

N = (Total buckling load on stiffened plate)/b (lb/in.)

N_{eq} = Buckling load per unit width on a metal plate of same mass as composite reinforced plate

FIGURE 5.6. Simply Supported Plate with Single Eccentric Shallow Stiffener

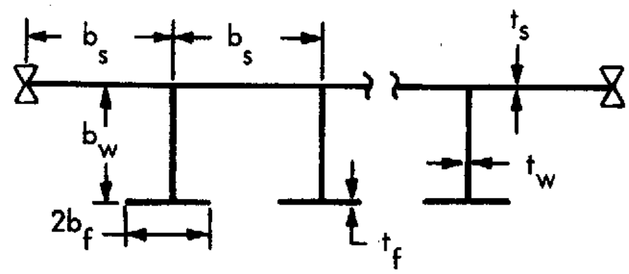
(Test Problem 3)



(a) Deep Stiffener (Case B of Figure 5.5)

(b) Shallow Stiffener (Case A of Table 5.2)

FIGURE 5.7. Simply Supported Plate with Single Eccentric Deep and Shallow Stiffeners - Buckling Mode Shape (Test Problem 3)



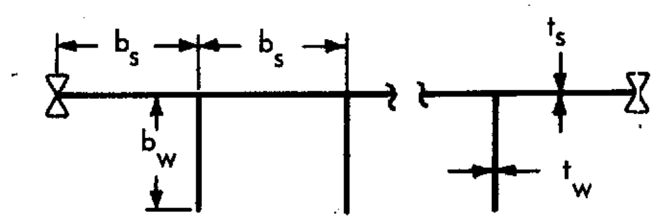
A. Tee Section Stiffened Plate

b_s = 3.0 ins. t_s = 0.080 ins.

b_w = 1.8 ins. t_w = 0.056 ins.

b_f = 0.54 ins. $t_f = t_w$

No. of stiffeners = 6



B. Integrally Stiffened Plate

b_s = 2.05 ins. t_s = 0.089 ins.

b_w = 1.06 ins. t_w = 0.058 ins.

No. of stiffeners = 6

Material Properties:

Aluminum Alloy

$E_{11} = E_{22} = 9.5 \times 10^6$ psi

$G_{12} = G_{23} = 3.655 \times 10^6$ psi

$\nu_{12} = 0.3$

FIGURE 5.8. T-Section and Integrally Stiffened Plates - Data (Test Problem 4)

Plate No.	Length (ins.)	Results from Program			k_s^* from Ref. 4 of Appendix B
		Critical Stress σ_{cr} psi	k_s^*	Axial half-wave No. m	
A	15.0	25800	4.25	5	4.30
B	12.3	29950	1.86	6	1.87

$$*\sigma_{cr} = \frac{k_s \pi^2 E_{11}}{12(1 - \nu_{12}^2)}$$

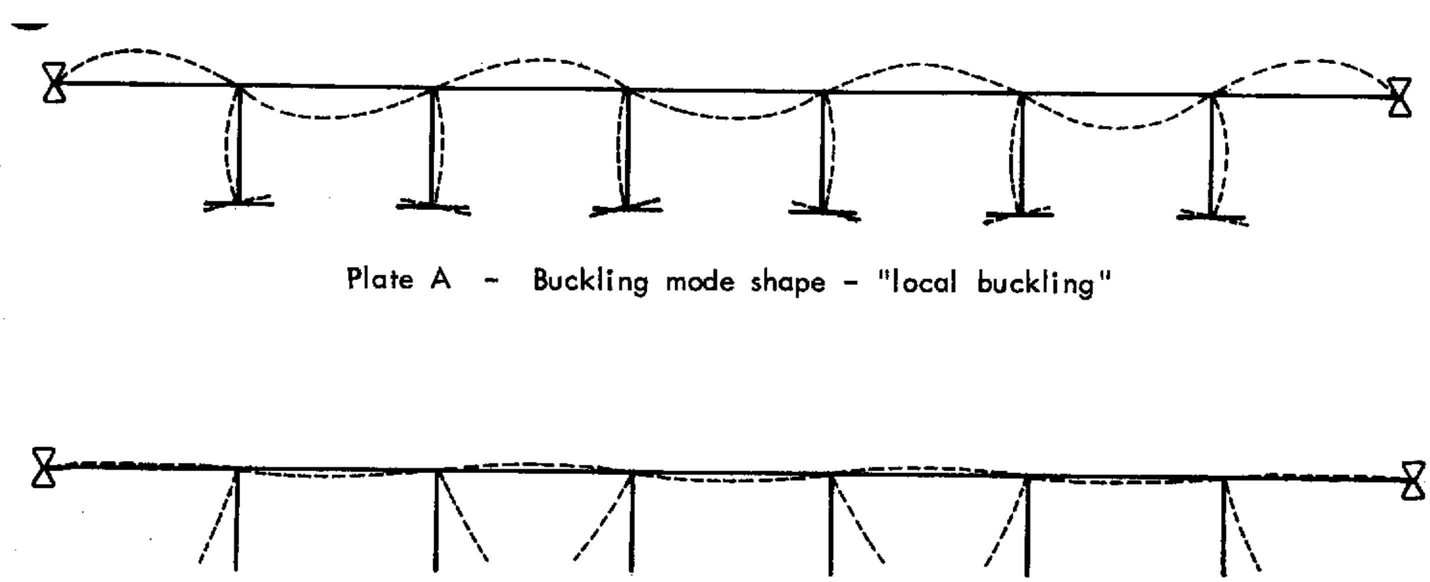
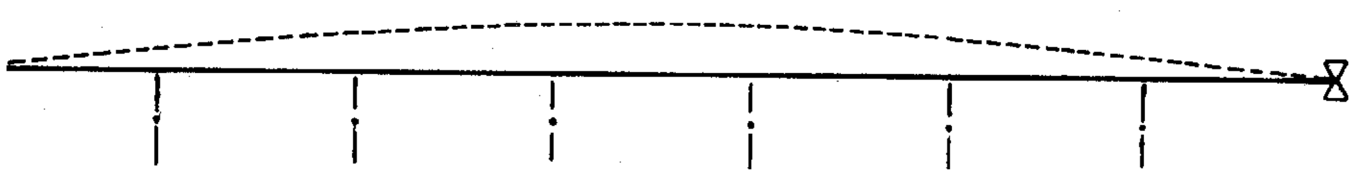


Plate A - Buckling mode shape - "local buckling"

Plate B - Buckling mode shape - "local buckling"

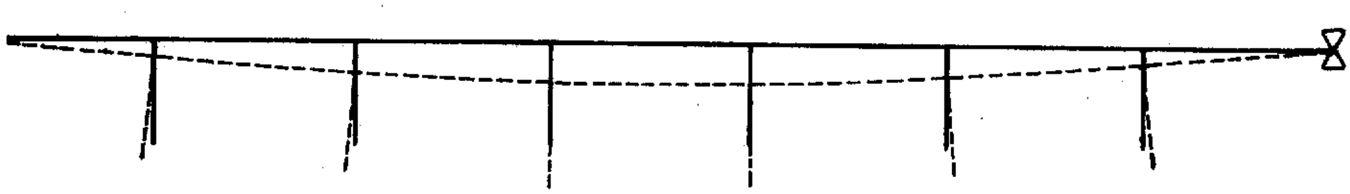
FIGURE 5.9. T-Section and Integrally Stiffened Plates - "Local Buckling" Results

Plate Number (see Fig. 5.8)	Length (ins.)	Results from Program		Results from Reference 5 of Appendix B	
		Critical Load P (lbs.)	Axial half-wave no. m	Critical Load P (lbs.)	Axial half-wave no. m
B (Stiffeners idealized as beam elements)	25.0	22432	1	23450	1
B (Stiffeners idealized as flat plate elements)	25.0	29470	1	-	-



Stiffeners idealized as beam elements

(NOTE: Beam element displacements omitted for clarity)



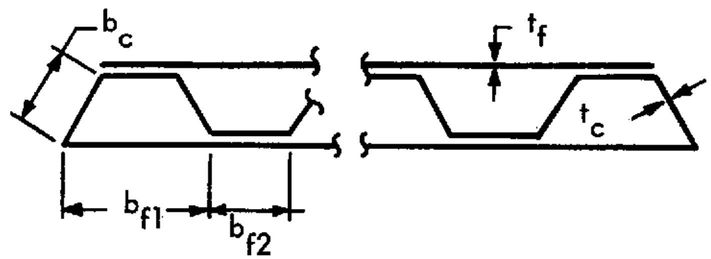
Stiffeners idealized as plate elements

FIGURE 5.10. Integrally Stiffened Plates - "General Instability" Results

(Test Problem 4)

(NOTE: All dimensions in inches.)

Test Problem 5



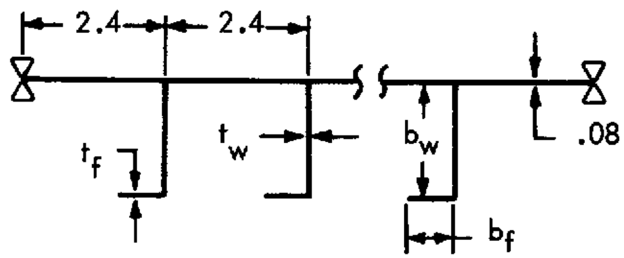
Aluminum alloy — 7 cells

$a = 8.96$	$t_c = .036$
$b_{f1} = 2.56$	$t_f = .048$
$b_{f2} = 1.06$	
$b_c = 1.06$	

Test Problem 6A

Test Problem 6B

Test Problem 6C



Aluminum alloy — 6 stiffeners

	a	b_w	b_f	$t_w = t_f$
6A	14.4	1.92	.576	.04
6B	14.4	0.96	.288	.04
6C	16.0	1.50	.500	.048

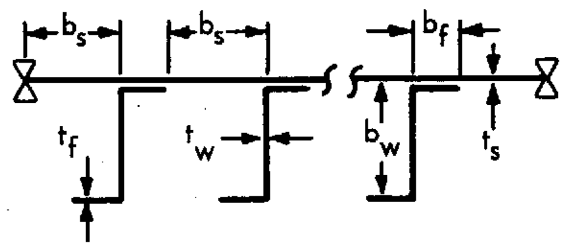
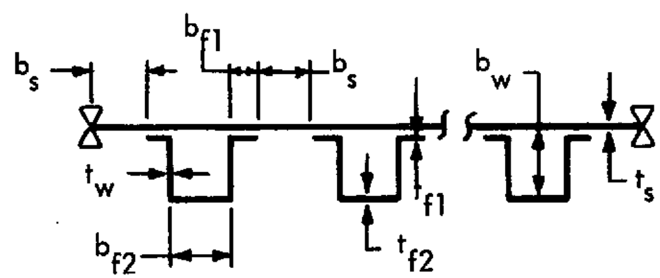
a = Axial Length

Material Properties:

$E_{11} = E_{22} = 10.0 \times 10^6$ psi $\quad G_{12} = 3.85 \times 10^6$ psi $\quad \nu_{12} = 0.3$

FIGURE 5.11. Additional Test Problems - Data

(NOTE: All dimensions in inches.)

Test Problem 7		Aluminum alloy 6 stiffeners $a = 16.0$ $b_s = 1.876$ $t_s = .080$ $b_w = 1.436$ $t_w = t_f = .048$ $b_f = 0.500$
Test Problem 8		Aluminum alloy 5 stiffeners $a = 20.0$ $b_s = 1.5$ $t_s = .08$ $b_w = 1.436$ $t_w = t_{f1} = t_{f2} = .048$ $b_{f1} = 0.5$ $b_{f2} = 1.2$

a = Axial length

Material Properties:

$E_{11} = E_{22} = 10.0 \times 10^6$ psi $G_{12} = 3.85 \times 10^6$ $\nu_{12} = 0.3$

FIGURE 5.11. Additional Test Problems - continued

TABLE 5.4

Additional Test Problems - Results

Plate No.	Results from Program		Results from Literature		Remarks
	Buckling Stress σ_{cr} psi	Axial half-wave no. m	Buckling Stress σ_{cr} psi	Axial half-wave no. m	
Test Problem 5	19550	5	20400	5	[1]
Test Problem 6A	21300	9	21400	-	[2]
Test Problem 6B	41600	6	42000	-	[2]
Test Problem 6C	41900	7	41200	-	[2]
Test Problem 7	49800	12	40900	7	[1]
Test Problem 8	52500	17	47700	10	[1]

[1] Engineering Sciences Data 02.01.35 to 02.01.37, Engineering Sciences Data Unit, Royal Aeronautical Society, London.

[2] Becker, Herbert: Handbook of Structural Stability, Part II, TN 3782, NACA, July, 1957, Figure 14.

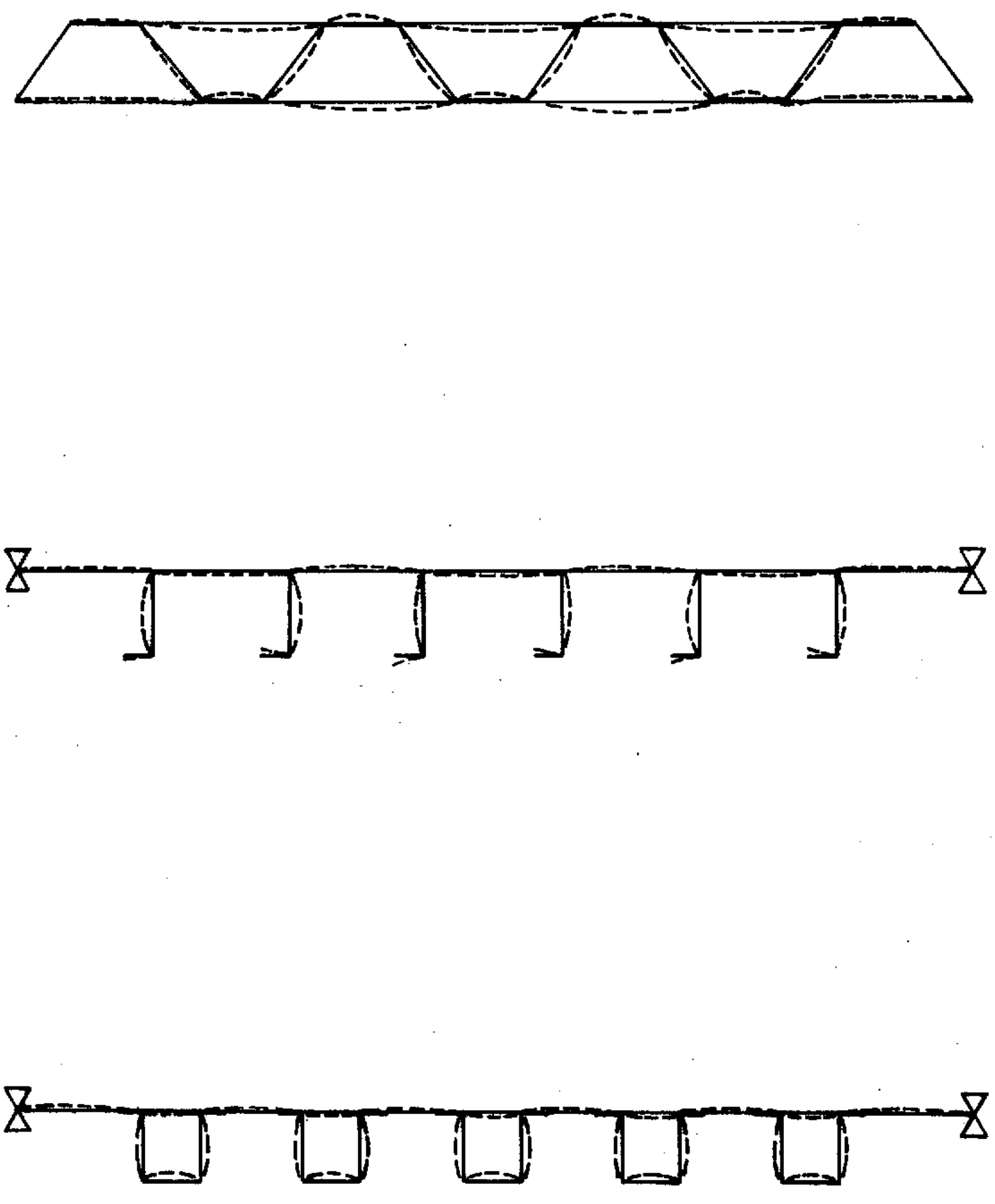


FIGURE 5.12. <u>Buckling Mode Shapes for Test Problems 5, 6, and 8</u>

5.2 Functional Test of Program

The program logic is tested by running various data sets such that all major logical paths are tested. The testing is divided into three categories:

(a) Functional Tests
(b) Inspection
(c) Engineering Tests (as shown in Section 5.1)

For the purpose of checking the logic of the main program DATAPRO and the routine MACON, several runs are made where the various input options as described in the data input specifications are tested. Specific checkout for subroutine BLKDET, CDTM, ZARK, BLU, and FBSUB is given in the Program Description Document under their respective subroutine descriptions.

For subroutines where the logic is considered trivial, checking is done by inspection. For subroutines DB and DBGENS the program has also been checked through inspection of the intermediate results that are optionally printed out.

Engineering tests shown in the previous two sections serve the purpose of functional tests for all subroutines and the three main programs, as the data included there covers the remaining logical design of the program. The degree to which this data is shown to give correct results is also indicative of the correctness of the program logic.

The test procedures for the various routines are given in Table 5.7.

The items that are tested in the various data sets are shown in Tables 5.5 and 5.6.

Data for engineering tests is given in Section 5.1. The data for the functional tests is available upon request.

TABLE 5.5 Engineering Test Data as Functional Tests

Item Name 1	Comment	Data Set			
		No. 1	No. 2	No. 3	No. 4
JPC(1) = 20	Corrugated core sandwich	Test 5			
JPC(1) = 30	Truss core sandwich	Test 2A	Test 2B		
JPC(1) = 50	Integral Panel	Test 4B			
JPC(1) = 60	Integral Panel with Tee-stiffeners	Test 4A			
JPC(1) = 70	Zee Stiffener Panel	Test 7	Test 6A	Test 6B	Test 6C
JPC(1) = 80	Hat Stiffener Panel	Test 8			
JPC(4) = 1	Read in type matrix for buck. element	Test 1	Test 3A	Test 3B	Test 3C
JPC(4) = 0	Type matrix set up by program	Test 5	Test 2A	Test 4B	Test 4A
JPC(3) = 0	Load only	ALL CASES			
JPC(3) = 1	Eigenvector	TESTS 1 to 8			
IBCOT = 1	Simply Supported Edge	Test 3A	Test 3B		
IBCOT = -XX	Beam Element Location	Test 3C	Test 3D	Test 3E	Test 3F

1 See Section 4.7 for Data Input Specs

TABLE 5.6 Functional Tests

Item Name 1	Comment	Data Set No. 1	No. 2	No. 3	No. 4
JPC(1) = 40	Honeycomb Core Sandwich	11E	11F		
JPC(1) = 10	Corrugated Plate	CORP1			
JPC(1) = 81	Hat Stiffeners with location reinforce.	11C	11D		
JPC(1) = 90	Angle stiffeners with loc. reinforce. I	11AX			
JPC(1) = 91	Angle stiffeners with loc. reinforce. II	11A	11B		
JPC(1) = 100	Plate	OPL3	IPL3		
JPC(2) = 1	No. of diagonal partitions	OPLX1	(Partially checked out)		
JPC(2) = 2	"	OPLX2	(Partially checked out)		
JPC(2) = 3 or more	"	OPLX3	OPL3	IPL3	
IPC(1) = 1	Intermediate Results	QPLY3			
IPC(2) = 1	"	QPLY3			
IPC(16) = 1	Interrupt after data preprocessing	QPLY5			
CO = 0	Read engineering constants	QPLY1			
CO = 1	Read fiber and matrix properties	QPLY2			
CO = 2	Contiguity factors	QPLY2			
CO = 3	Read Q-matrix	QPLY4			

1 See Section 4.7 for Data Input Specs

TABLE 5.6 - continued

Item Name	Comment	Data Set			
		No. 1	No. 2	No. 3	No. 4
CO2 = 0	General Beam Element	CL3C			
CO2 = 1	Rect. Beam Element Laminated	CL3B			
CO2 = 1	Rect. Beam Element turned 90°	CL3A			
CO2 = 2	Circular Beam Element Laminated	CB1A			
MOPT = 0	Program sets range of modes	QPLY2			
MOPT = 1	User sets lower mode limit	QPLY4			
MOPT = 3	Alternate loop on modes	ZEEP1	(Partially	checked out)	
MOPT = 2	User sets range of modes	QPLY1			

TABLE 5.7

Functional Test Procedures for Program Subroutines

Subroutine or Program Name	Purpose	Tests
BUCLASP	Main Program Driver	Engineering and functional test data, inspection
DATAPRO	Main Program Data Preprocessor	Engineering and functional tests, inspection of out formats
MACON	Material Constants	Logic trivial, functional test data, inspection
LOADING	Main Program Load Solution	Engineering and functional tests
DB	Buckling Determinant	Engineering test data, inspection of coding, inspection of intermediate results
DT	Equilibrium Equations	Logic trivial, engineering tests, inspection of coding
RGEN	Coefficients for Equilibrium Equations	Logic trivial, engineering tests, inspection of coding
DBGENS	Generates Elements of Buckling Determinant	Logic trivial, engineering tests, inspection of coding, inspection of intermediate results
BLKDET, ELIM MATZ, TRANSI	Real Determinant	See Section 3.6, Program Description Document
CDTM	Complex Determinant	See Section 3.7, Program Description Document

TABLE 5.7 - continued

Functional Test Procedures for Program Subroutines

Subroutine or Program Name	Purpose	Tests
ZARK	Complex Root Finder	See Section 3.8, Program Description Document.
DETZER	Interpolation	See Section 3.9, Program Description Document.
DISPLAC	Main Program Eigenvectors and Displacement	Engineering and functional tests, temporary intermediate print
BANDW	Find bandwidth of buckling determinant	Engineering and functional tests inspection, temporary print
COMPAC	Transfer buckling determinant to compact form	Engineering and function tests, print of determinant value after BLU routine
TURN	Transfer one row to compact form	Temporary intermediate print, inspection
EIGV	Eigenvector Solution	Engineering and functional tests, convergence checks
BLU	Decomposition	See Section 3.14, Program Description Document.
FBSUB	Forward/Backward Substitution	See Section 3.15, Program Description Document
DIS	Relative Displacements	Engineering and functional tests, inspection of consistant displacements.

6.0 SAMPLE PROBLEM

6.1 Input for Sample Problem

```
 TEST PANEL TYPE NO 5   INTEGRAL PANEL  5 STIFFENERS

50    5    1
5     8    2
      7         0         8      12.3      2000.       50.        5.
   0.0      -2.05
   0.0       0.0
 1.055       0.0
   0.0       2.05
 1.055       2.05
   0.0       4.10
 1.055       4.10
   0.0       6.15
 1    2    1
 2    3    1
 2    4    1
 4    5    1
 4    6    1
 6    7    1
 6    8    1
 2
 1    1
 8    1
      1
 .089   9.5+6              .3 3.655+6
      1
 .058   9.5+6              .3 3.655+6
      1
 .089   9.5+6              .3 3.655+6
      1
 .058   9.5+6              .3 3.655+6
      1
 .089   9.5+6              .3 3.655+6
      1
 .058   9.5+6              .3 3.655+6
      1
 .089   9.5+6              .3 3.655+6
```

6.2 Output for Sample Problem

```
PROGRAM S0325A/BUCLASP  CERTIFIED 11/20/70                              NOV 23  70

BUCKLING LOADS OF ORTHOTROPIC LAMINATED STIFFENED PLATES

LOADING              -- UNIAXIAL COMPRESSION
BOUNDARY CONDITIONS -- LOADED EDGES ARE SIMPLY SUPPORTED, UNLOADED SIDES
ARE FREE, SIMPLY SUPP., CLAMPED, OR SUPPORTED BY BEAM ELEMENT
BEAM ELEMENT HAS SIMPLY SUPP. ENDS AT ITS NEUTRAL AXIS

********************************************************************************

   TEST PANEL TYPE NO 5   INTEGRAL PANEL  5 STIFFENERS

    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
   50    5    1   -0

 SECTION TYPE       50

 NMI =     5
 NMA =     8
 NOPT=     2

NUMBER OF ELEMENTS    LL    =              7
NUMBER OF BEAM EL.    LB    =              0
NUMBER OF NODES       NOD   =              8

LENGTH                AL    =         12.300

STARTING LOAD         STLD  =       2000.000
PRIMARY INTERVAL      SINC  =         50.000
SEQ. INTERVAL         SINC2 =          5.000

NODAL COORDINATES

      NODE       Z         Y
         1    0.0000   -2.0500
         2    0.0000    0.0000
         3    1.0550    0.0000
         4    0.0000    2.0500
         5    1.0550    2.0500
         6    0.0000    4.1000
         7    1.0550    4.1000
         8    0.0000    6.1500

ELEMENT DATA
ELEMENT NO     NODE I    NODE J
         1         1         2         1
         2         2         3         1
         3         2         4         1
         4         4         5         1
         5         4         6         1
         6         6         7         1
         7         6         8         1

BOUNDARY CONDITIONS
      NODE      CODE
```

1	1
2	0
3	0
4	0
5	0
6	0
7	0
8	1

ELEMENT NUMBER 1 FLAT PLATE

NUMBER OF LAYERS LA = 1

LAYER NO 1 INPUT OPTION NO 0 WAS USED

THE MATERIAL PROPERTIES WAS ENTERED AS E11,E22 ETC.

THICKNESS	T1 =	.0890
E-MODULUS	E1 =	9500000.0000
	E2 =	9500000.0000
POISSONS RATIO	RNUA=	.3000
	RNUB=	.3000
TORSIONAL MOD.	G12 =	3655000.0000

LAYER	EXX	EYY	MUXY	MUYX	G
1	9.500000E+06	9.500000E+06	3.000000E-01	3.000000E-01	3.655000E+06

Q-MATRIX LAYER NO 1

10439560.440	3131868.132	0.000
3131868.132	10439560.440	0.000
0.000	0.000	3655000.000

LOCATION OF NEUTRAL PLANE
RELATIVE TO REFERENCE PLANE .0445

A-MATRIX

929120.879	278736.264	0.000
278736.264	929120.879	0.000
0.000	0.000	325295.000

B-MATRIX

.000	.000	0.000
.000	.000	0.000
0.000	0.000	.000

D-MATRIX

613.297	183.989	0.000
183.989	613.297	0.000
0.000	0.000	214.722

ELEMENT NUMBER 2 FLAT PLATE

NUMBER OF LAYERS LA = 1

LAYER NO 1 INPUT OPTION NO 0 WAS USED

THE MATERIAL PROPERTIES WAS ENTERED AS E11,E22 ETC.

THICKNESS	T1 =	.0580
E-MODULUS	E1 =	9500000.0000
	E2 =	9500000.0000
POISSONS RATIO	RNUA=	.3000
	RNUB=	.3000
TORSIONAL MOD.	G12 =	3655000.0000

LAYER	EXX	EYY	MUXY	MUYX	G
1	9.500000E+06	9.500000E+06	3.000000E-01	3.000000E-01	3.655000E+06

Q-MATRIX LAYER NO 1

10439560.440	3131868.132	0.000
3131868.132	10439560.440	0.000
0.000	0.000	3655000.000

LOCATION OF NEUTRAL PLANE
RELATIVE TO REFERENCE PLANE .0290

A-MATRIX

605494.505	181648.352	0.000
181648.352	605494.505	0.000
0.000	0.000	211990.000

B-MATRIX

.000	.000	0.000
.000	.000	0.000
0.000	0.000	.000

D-MATRIX

169.740	50.922	0.000
50.922	169.740	0.000
0.000	0.000	59.428

ELEMENT NUMBER 3 FLAT PLATE

NUMBER OF LAYERS LA = 1

LAYER NO 1 INPUT OPTION NO 0 WAS USED

THE MATERIAL PROPERTIES WAS ENTERED AS E11,E22 ETC.

THICKNESS	T1 =	.0890
E-MODULUS	E1 =	9500000.0000
	E2 =	9500000.0000
POISSONS RATIO	RNUA=	.3000
	RNUB=	.3000
TORSIONAL MOD.	G12 =	3655000.0000

LAYER	EXX	EYY	MUXY	MUYX	G
1	9.500000E+06	9.500000E+06	3.000000E-01	3.000000E-01	3.655000E+06

Q-MATRIX LAYER NO 1

10439560.440	3131868.132	0.000
3131868.132	10439560.440	0.000
0.000	0.000	3655000.000

LOCATION OF NEUTRAL PLANE
RELATIVE TO REFERENCE PLANE .0445

A-MATRIX

929120.879	278736.264	0.000
278736.264	929120.879	0.000
0.000	0.000	325295.000

B-MATRIX

.000	.000	0.000
.000	.000	0.000
0.000	0.000	.000

D-MATRIX

613.297	183.989	0.000
183.989	613.297	0.000
0.000	0.000	214.722

ELEMENT NUMBER 4 FLAT PLATE

NUMBER OF LAYERS LA = 1

LAYER NO 1 INPUT OPTION NO 0 WAS USED

THE MATERIAL PROPERTIES WAS ENTERED AS E11,E22 ETC.

THICKNESS	T1 =	.0580
E-MODULUS	E1 =	9500000.0000
	E2 =	9500000.0000
POISSONS RATIO	RNUA=	.3000
	RNUB=	.3000
TORSIONAL MOD.	G12 =	3655000.0000

LAYER	EXX	EYY	MUXY	MUYX	G
1	9.500000E+06	9.500000E+06	3.000000E-01	3.000000E-01	3.655000E+06

Q-MATRIX LAYER NO 1

10439560.440	3131868.132	0.000
3131868.132	10439560.440	0.000
0.000	0.000	3655000.000

LOCATION OF NEUTRAL PLANE
RELATIVE TO REFERENCE PLANE .0290

A-MATRIX

605494.505	181648.352	0.000
181648.352	605494.505	0.000
0.000	0.000	211990.000

B-MATRIX

.000	.000	0.000
.000	.000	0.000
0.000	0.000	.000

D-MATRIX

169.740	50.922	0.000
50.922	169.740	0.000
0.000	0.000	59.428

ELEMENT NUMBER 5 FLAT PLATE

NUMBER OF LAYERS LA = 1

LAYER NO 1 INPUT OPTION NO 0 WAS USED

THE MATERIAL PROPERTIES WAS ENTERED AS E11,E22 ETC.

THICKNESS	T1 =	.0890
E-MODULUS	E1 =	9500000.0000
	E2 =	9500000.0000
POISSONS RATIO	RNUA=	.3000
	RNUB=	.3000
TORSIONAL MOD.	G12 =	3655000.0000

LAYER	EXX	EYY	MUXY	MUYX	G
1	9.500000E+06	9.500000E+06	3.000000E-01	3.000000E-01	3.655000E+06

Q-MATRIX LAYER NO 1

10439560.440	3131868.132	0.000
3131868.132	10439560.440	0.000
0.000	0.000	3655000.000

LOCATION OF NEUTRAL PLANE
RELATIVE TO REFERENCE PLANE .0445

A-MATRIX

929120.879	278736.264	0.000
278736.264	929120.879	0.000
0.000	0.000	325295.000

B-MATRIX

.000	.000	0.000
.000	.000	0.000
0.000	0.000	.000

D-MATRIX

613.297	183.989	0.000
183.989	613.297	0.000
0.000	0.000	214.722

ELEMENT NUMBER 6 FLAT PLATE

NUMBER OF LAYERS LA = 1

LAYER NO 1 INPUT OPTION NO 0 WAS USED

THE MATERIAL PROPERTIES WAS ENTERED AS E11,E22 ETC.

THICKNESS	T1	=	.0580
E-MODULUS	E1	=	9500000.0000
	E2	=	9500000.0000
POISSONS RATIO	RNUA=		.3000
	RNUB=		.3000
TORSIONAL MOD.	G12	=	3655000.0000

LAYER	EXX	EYY	MUXY	MUYX	G
1	9.500000E+06	9.500000E+06	3.000000E-01	3.000000E-01	3.655000E+06

Q-MATRIX LAYER NO 1

10439560.440	3131868.132	0.000
3131868.132	10439560.440	0.000
0.000	0.000	3655000.000

LOCATION OF NEUTRAL PLANE
RELATIVE TO REFERENCE PLANE .0290

A-MATRIX

605494.505	181648.352	0.000
181648.352	605494.505	0.000
0.000	0.000	211990.000

B-MATRIX

.000	.000	0.000
.000	.000	0.000
0.000	0.000	.000

D-MATRIX

169.740	50.922	0.000
50.922	169.740	0.000
0.000	0.000	59.428

ELEMENT NUMBER 7 FLAT PLATE

NUMBER OF LAYERS LA = 1

LAYER NO 1 INPUT OPTION NO 0 WAS USED

THE MATERIAL PROPERTIES WAS ENTERED AS E11,E22 ETC.

THICKNESS	T1 =	.0890
E-MODULUS	E1 =	9500000.0000
	E2 =	9500000.0000
POISSONS RATIO	RNUA=	.3000
	RNUB=	.3000
TORSIONAL MOD.	G12 =	3655000.0000

LAYER	EXX	EYY	MUXY	MUYX	G
1	9.500000E+06	9.500000E+06	3.000000E-01	3.000000E-01	3.655000E+06

Q-MATRIX LAYER NO 1

10439560.440	3131868.132	0.000
3131868.132	10439560.440	0.000
0.000	0.000	3655000.000

LOCATION OF NEUTRAL PLANE
RELATIVE TO REFERENCE PLANE .0445

A-MATRIX

929120.879	278736.264	0.000
278736.264	929120.879	0.000
0.000	0.000	325295.000

B-MATRIX

.000	.000	0.000
.000	.000	0.000
0.000	0.000	.000

D-MATRIX

613.297	183.989	0.000
183.989	613.297	0.000
0.000	0.000	214.722

SUMMARY OF TYPES OF PLATE ELEMENTS

ELEM. NO.	TYPE NO.
1	1
2	2
3	1
4	2
5	1
6	2
7	1

NUMBER OF NON-STANDARD OFFSETS -0

ELEM. NO.	WIDTH	TRANSFORMATION SIN	COS	EL. TYPE
1	2.05000	0.00000	1.00000	FLAT PLATE
2	1.05500	1.00000	0.00000	FLAT PLATE
3	2.05000	0.00000	1.00000	FLAT PLATE
4	1.05500	1.00000	0.00000	FLAT PLATE
5	2.05000	0.00000	1.00000	FLAT PLATE
6	1.05500	1.00000	0.00000	FLAT PLATE
7	2.05000	0.00000	1.00000	FLAT PLATE

NET OFFSETS

ELEM. NO.	START ZO	START YO	END ZO	END YO	ZNX

1	.00000	0.00000	.00000	0.00000	.04450
2	.00000	0.00000	.00000	0.00000	.02900
3	.00000	0.00000	.00000	0.00000	.04450
4	.00000	0.00000	.00000	0.00000	.02900
5	.00000	0.00000	.00000	0.00000	.04450
6	.00000	0.00000	.00000	0.00000	.02900
7	.00000	0.00000	.00000	0.00000	.04450

ITYPA-MATRIX

11	0	0
3	1	0
3	0	1
4	2	2
0	4	0

ITYPM-MATRIX

3	1	0
3	0	1
4	2	2
0	4	0

ITYPB-MATRIX

3	1	0
3	0	1
4	2	2
0	4	0
0	0	13

ARRAY SPACE REQUIREMENTS FOR BUCKLING DET. BLOCKS ARE

DBMA-MATRIX(+SCRATCH SPACE)	672 DEC.	0000001240 OCT.
DBM -MATRIX	384 DEC.	0000000600 OCT.
DBMB-MATRIX	480 DEC.	0000000740 OCT.
SCRATCH ARRAYS	64 DEC.	0000000100 OCT.
TOTAL ARRAY SPACE(BLANK COMMON)	1600 DEC.	0000003100 OCT.

```
************************************************

ASSUMING A PROGRAM LENGTH OF       0000050000  OCT.

(EXCLUDING BLANK COMMON)

FOR LOAD CALCULATION THE
RECOMMENDED FIELD LENGTH IS        0000053100  OCT.

************************************************

TIME FOR DATA INPUT AND PREPROCESSING       .414   CP-SEC.
```

```
******************************
*                            *
*    MODE   IS   M =    5    *
*                            *
*                            *
******************************
```

M = 5

LOAD ON FIRST EL. (PLI)	TOTAL LOAD (POUNDS)	DB-VALUES (BUC. DET)	COMMENTS	
2000.0000000000	31475.281	-7.45516E-01		712
2050.0000000000	32262.163	-8.71262E-01		712
2100.0000000000	33049.045	-9.85128E-01		712
2150.0000000000	33835.927	-6.72766E-02		716
2200.0000000000	34622.809	-7.09164E-02		716
2250.0000000000	35409.691	-7.19374E-02		716
2300.0000000000	36196.573	-6.99887E-02		716
2350.0000000000	36983.455	-6.50101E-02		716
2400.0000000000	37770.337	-9.16685E-01		712
2450.0000000000	38557.219	-7.59863E-01		712
2500.0000000000	39344.101	-5.85140E-01		712
2550.0000000000	40130.983	-4.10919E-01		712
2600.0000000000	40917.865	-2.55528E-01		712
2650.0000000000	41704.747	-1.33600E-01		712
2700.0000000000	42491.629	-8.42791E-01		708
2750.0000000000	43278.511	-1.78043E-01		708
2800.0000000000	44065.393	3.54537E-01	SIGN CHANGE IN DB	704
2755.0000000000	43357.199	-1.40829E-01		708
2760.0000000000	43435.888	-1.08100E-01		708
2765.0000000000	43514.576	-7.96171E-02		708
2770.0000000000	43593.264	-8.82139E-01		704
2775.0000000000	43671.952	-5.50364E-01		704
2780.0000000000	43750.640	-2.74440E-01		704
2785.0000000000	43829.329	-8.03204E-01		700
2790.0000000000	43908.017	1.26560E-01	SIGN CHANGE IN DB	704
2787.5000000000	43868.673	7.01617E-01	SIGN CHANGE IN DB	700
2786.3343848181	43850.329	3.69521E-01		696
2786.2947104498	43849.704	-1.80343E-01		692
2786.2958848077	43849.723	7.03582E-01		680
2786.2958569576	43849.722	-6.57445E-02		688

ELEMENT LOADS MODE M 5

ELEM. NO.	LINE LOAD (PLATE EL. ONLY)		TOTAL ELEMENT LOAD		ELEMENT TYPE
1	2786.2959	LBS/INCH	5711.9066	LBS	FLAT PLATE
2	1815.7883	LBS/INCH	1915.6567	LBS	FLAT PLATE
3	2786.2959	LBS/INCH	5711.9066	LBS	FLAT PLATE
4	1815.7883	LBS/INCH	1915.6567	LBS	FLAT PLATE
5	2786.2959	LBS/INCH	5711.9066	LBS	FLAT PLATE
6	1815.7883	LBS/INCH	1915.6567	LBS	FLAT PLATE
7	2786.2959	LBS/INCH	5711.9066	LBS	FLAT PLATE

AXIAL STRAIN IS 3.29544E-03

CRITICAL LOAD

```
LINE LOAD ON EL. ONE =          2786.296 P.L.I.
TOTAL LOAD           =         43849.723 POUNDS
MODE M =      5

   TEST PANEL TYPE NO 5   INTEGRAL PANEL  5 STIFFENERS


TIMING
------


FOR MODE M  =      5
EXECUTION TIME IS                 11.180    CP-SECONDS
```

```
******************************
*                            *
*    MODE   IS   M =    6    *
*                            *
*                            *
******************************
```

M = 6

LOAD ON FIRST EL. (PLI)	TOTAL LOAD (POUNDS)	DB-VALUES (BUC. DET)	COMMENTS	
2000.0000000000	31475.281	-1.38077E-01		748
2050.0000000000	32262.163	-1.68501E-01		748
2100.0000000000	33049.045	-1.94337E-01		748
2150.0000000000	33835.927	-2.11650E-01		748
2200.0000000000	34622.809	-2.17090E-01		748
2250.0000000000	35409.691	-2.08715E-01		748
2300.0000000000	36196.573	-1.86682E-01		748
2350.0000000000	36983.455	-1.53584E-01		748
2400.0000000000	37770.337	-1.14233E-01		748
2450.0000000000	38557.219	-7.47589E-02		748
2500.0000000000	39344.101	-6.58282E-01		744
2550.0000000000	40130.983	-2.80031E-01		744
2600.0000000000	40917.865	-7.60359E-02		744
2650.0000000000	41704.747	-1.00403E-01		740
2700.0000000000	42491.629	9.20758E-02	SIGN CHANGE IN DB	736
2655.0000000000	41783.435	-6.37539E-02		740
2660.0000000000	41862.124	-5.72545E-01		736
2665.0000000000	41940.812	-2.45023E-01		736
2670.0000000000	42019.500	-3.08983E-01		732
2675.0000000000	42098.188	1.21933E-01	SIGN CHANGE IN DB	736
2672.5000000000	42058.844	9.73156E-01	SIGN CHANGE IN DB	732
2670.6024756708	42028.982	4.73642E-01		728
2670.5498011102	42028.153	1.97129E-01		724
2670.5483943205	42028.130	-1.31595E-01		716
2670.5484210205	42028.131	8.28764E-01		716

ELEMENT LOADS MODE M 6

ELEM. NO.	LINE LOAD (PLATE EL. ONLY)		TOTAL ELEMENT LOAD		ELEMENT TYPE
1	2670.5484	LBS/INCH	5474.6242	LBS	FLAT PLATE
2	1740.3574	LBS/INCH	1836.0770	LBS	FLAT PLATE
3	2670.5484	LBS/INCH	5474.6242	LBS	FLAT PLATE
4	1740.3574	LBS/INCH	1836.0770	LBS	FLAT PLATE
5	2670.5484	LBS/INCH	5474.6242	LBS	FLAT PLATE
6	1740.3574	LBS/INCH	1836.0770	LBS	FLAT PLATE
7	2670.5484	LBS/INCH	5474.6242	LBS	FLAT PLATE

AXIAL STRAIN IS 3.15854E-03

CRITICAL LOAD

LINE LOAD ON EL. ONE = 2670.548 P.L.I.
TOTAL LOAD = 42028.130 POUNDS
MODE M = 6

TEST PANEL TYPE NO 5   INTEGRAL PANEL 5 STIFFENERS

TIMING

```
FOR MODE M  =      6
EXECUTION TIME IS                  9.320    CP-SECONDS
```

1

```
*****************************
*                           *
*    MODE   IS   M =    7   *
*                           *
*                           *
*****************************
```

M = 7

LOAD ON FIRST EL. (PLI)	TOTAL LOAD (POUNDS)	DB-VALUES (BUC. DET)	COMMENTS	
2000.0000000000	31475.281	-9.74882E-02		772
2050.0000000000	32262.163	-6.71851E-02		776
2100.0000000000	33049.045	-2.32494E-01		776
2150.0000000000	33835.927	-5.10982E-01		776
2200.0000000000	34622.809	-8.63243E-01		776
2250.0000000000	35409.691	-7.55946E-02		780
2300.0000000000	36196.573	-9.08531E-02		780
2350.0000000000	36983.455	-9.48193E-02		780
2400.0000000000	37770.337	-8.57188E-02		780
2450.0000000000	38557.219	-6.59859E-02		780
2500.0000000000	39344.101	-6.66689E-01		776
2550.0000000000	40130.983	-3.20654E-01		776
2600.0000000000	40917.865	-1.00021E-01		776
2650.0000000000	41704.747	-2.04431E-01		772
2700.0000000000	42491.629	3.96171E-01	SIGN CHANGE IN DB	764
2655.0000000000	41783.435	-1.50724E-01		772
2660.0000000000	41862.124	-1.07651E-01		772
2665.0000000000	41940.812	-7.39055E-02		772
2670.0000000000	42019.500	-7.71296E-01		768
2675.0000000000	42098.188	-4.69011E-01		768
2680.0000000000	42176.876	-2.56724E-01		768
2685.0000000000	42255.565	-1.16702E-01		768
2690.0000000000	42334.253	-5.21895E-01		764
2695.0000000000	42412.941	1.63765E-01	SIGN CHANGE IN DB	764
2692.5000000000	42373.597	-1.10215E-01		764
2695.0000000000	42412.941	1.63765E-01	SIGN CHANGE IN DB	764
2693.5056828346	42389.424	2.41819E-01		760
2693.3844048098	42387.515	2.87978E-01		756
2693.3746521913	42387.362	-8.58556E-01		748
2693.3747644608	42387.364	1.86780E-01		740
2693.3747375357	42387.363	-2.05347E-01		748

ELEMENT LOADS MODE M 7

ELEM. NO.	LINE LOAD (PLATE EL. ONLY)		TOTAL ELEMENT LOAD		ELEMENT TYPE
1	2693.3748	LBS/INCH	5521.4183	LBS	FLAT PLATE
2	1755.2330	LBS/INCH	1851.7708	LBS	FLAT PLATE
3	2693.3748	LBS/INCH	5521.4183	LBS	FLAT PLATE
4	1755.2330	LBS/INCH	1851.7708	LBS	FLAT PLATE
5	2693.3748	LBS/INCH	5521.4183	LBS	FLAT PLATE
6	1755.2330	LBS/INCH	1851.7708	LBS	FLAT PLATE
7	2693.3748	LBS/INCH	5521.4183	LBS	FLAT PLATE

AXIAL STRAIN IS 3.18554E-03

ICAL LOAD

```
LINE LOAD ON EL. ONE =          2693.375 P.L.I.
TOTAL LOAD           =         42387.364 POUNDS
MODE M =      7
```

TEST PANEL TYPE NO 5   INTEGRAL PANEL 5 STIFFENERS

TIMING

```
FOR MODE M  =      7
EXECUTION TIME IS              11.602    CP-SECONDS
```

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*****************************
*                           *
*     MODE   IS   M =   8   *
*                           *
*                           *
*****************************
```

SPOTCHECK 1	LOADING	2475.000	LBS/IN	BUCKL.DET.	-.207*2**	812
SPOTCHECK 2	LOADING	2525.000	LBS/IN	BUCKL.DET.	-.140*2**	808

LOAD ON FIRST EL. (PLI)	TOTAL LOAD (POUNDS)	DB-VALUES (BUC. DET)	COMMENTS	
2000.0000000000	31475.281	-7.14225E-02		824
2050.0000000000	32262.163	-8.77078E-01		820
2100.0000000000	33049.045	-6.31925E-01		820
2150.0000000000	33835.927	-4.23739E-01		820
2200.0000000000	34622.809	-2.61295E-01		820
2250.0000000000	35409.691	-1.45655E-01		820
2300.0000000000	36196.573	-7.15480E-02		820
2350.0000000000	36983.455	-4.75860E-01		816
2400.0000000000	37770.337	-1.55954E-01		816
2450.0000000000	38557.219	-5.57870E-01		812
2500.0000000000	39344.101	-9.13671E-01		808
2550.0000000000	40130.983	-6.87441E-01		800
2600.0000000000	40917.865	-2.57020E-01	DBLE ROOT ENCOUNTERD	800
2555.0000000000	40209.671	-9.45150E-02		800
2560.0000000000	40288.360	-4.29496E-01		788
2565.0000000000	40367.048	-1.60648E-01	DBLE ROOT ENCOUNTERD	792
2562.5000000000	40327.704	-2.15984E-01	DBLE ROOT ENCOUNTERD	788
2561.2500000000	40308.032	-1.44037E-01	DBLE ROOT ENCOUNTERD	784
2560.6250000000	40298.196	-7.13585E-02	DBLE ROOT FOUND	772

THE DOUBLE ROOT IN THE P-VALUES ARE IN THE FOLLOWING INTERVAL
WHICH WILL BE IGNORED IN THE SEARCH FOR THE CRITICAL LOAD

NXU = 2560.625
NXL = 2560.000

SPOTCHECK 1	LOADING	2563.125	LBS/IN	BUCKL.DET.	-.501*2**	788
SPOTCHECK 2	LOADING	2568.125	LBS/IN	BUCKL.DET.	-.766*2**	792
SPOTCHECK 1	LOADING	2780.625	LBS/IN	BUCKL.DET.	-.584*2**	796
SPOTCHECK 2	LOADING	2830.625	LBS/IN	BUCKL.DET.	-.503*2**	788

LOAD ON FIRST EL. (PLI)	TOTAL LOAD (POUNDS)	DB-VALUES (BUC. DET)	COMMENTS	
2560.6250000000	40298.196	-7.13585E-02		772
2565.6250000000	40376.884	-2.36980E-01		792
2570.6250000000	40455.572	-1.09015E-01		796
2575.6250000000	40534.260	-3.40042E-01		796
2580.6250000000	40612.948	-7.44653E-01		796
2585.6250000000	40691.637	-8.38589E-02		800
2590.6250000000	40770.325	-1.33439E-01		800
2595.6250000000	40849.013	-1.94723E-01		800
2600.6250000000	40927.701	-2.66515E-01		800
2605.6250000000	41006.389	-3.47123E-01		800
2655.6250000000	41793.271	-7.13216E-02		804
2705.6250000000	42580.153	-9.16060E-01		800
2755.6250000000	43367.035	-2.01035E-01		800
2805.6250000000	44153.917	-7.76307E-01		788
2855.6250000000	44940.800	-7.71566E-01	WAS SIGN CH. MISSED	788
2830.6250000000	44547.358	-5.02538E-01	SPOTCHECK 2	788

SUBDIVIDE PREVIOUS STEPS TO CHECK FOR TWO OR MORE ZERO-CROSSINGS

NXU = 2855.625
STLD1 = 2805.625

M = 8

LOAD ON FIRST EL. (PLI)	TOTAL LOAD (POUNDS)	DB-VALUES (BUC. DET)	COMMENTS	
2805.6250000000	44153.917	-7.76307E-01		788
2810.6250000000	44232.606	6.89601E-02	SIGN CHANGE IN DB	792
2806.1250000000	44161.786	-4.15144E-01		788
2806.6250000000	44169.655	-9.95473E-02		788
2807.1250000000	44177.524	1.73246E-01	SIGN CHANGE IN DB	788
2806.8074594451	44172.527	7.70601E-02		784
2806.7990391544	44172.394	5.06035E-01		776
2806.7988174760	44172.391	-1.68094E-01		768
2806.7988455423	44172.391	6.34965E-02		776

ELEMENT LOADS MODE M 8

ELEM. NO.	LINE LOAD (PLATE EL. ONLY)		TOTAL ELEMENT LOAD		ELEMENT TYPE
1	2806.7988	LBS/INCH	5753.9376	LBS	FLAT PLATE
2	1829.1498	LBS/INCH	1929.7530	LBS	FLAT PLATE
3	2806.7988	LBS/INCH	5753.9376	LBS	FLAT PLATE
4	1829.1498	LBS/INCH	1929.7530	LBS	FLAT PLATE
5	2806.7988	LBS/INCH	5753.9376	LBS	FLAT PLATE
6	1829.1498	LBS/INCH	1929.7530	LBS	FLAT PLATE
7	2806.7988	LBS/INCH	5753.9376	LBS	FLAT PLATE

AXIAL STRAIN IS 3.31969E-03

CRITICAL LOAD

LINE LOAD ON EL. ONE = 2806.799 P.L.I.
TOTAL LOAD = 44172.391 POUNDS
MODE M = 8

TEST PANEL TYPE NO 5 INTEGRAL PANEL 5 STIFFENERS

TIMING

FOR MODE M = 8
EXECUTION TIME IS 18.630 CP-SECONDS

PROGRAM 50325A/DUCLASP CERTIFIED 11/20/70 NOV 23 70

BUCKLING LOADS OF ORTHOTROPIC LAMINATED STIFFENED PLATES

LOADING -- UNIAXIAL COMPRESSION
BOUNDARY CONDITIONS -- LOADED EDGES ARE SIMPLY SUPPORTED, UNLOADED SIDES
ARE FREE, SIMPLY SUPP., CLAMPED, OR SUPPORTED BY BEAM ELEMENT
BEAM ELEMENT HAS SIMPLY SUPP. ENDS AT ITS NEUTRAL AXIS

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TEST PANEL TYPE NO 5 INTEGRAL PANEL 5 STIFFENERS

LOAD	M
43849.723	5
42028.130	6
42387.364	7
44172.391	8

ELEMENT LOADS MODE M 6

ELEM. NO.	LINE LOAD (PLATE EL. ONLY)		TOTAL ELEMENT LOAD		ELEMENT TYPE
1	2670.5484	LBS/INCH	5474.6242	LBS	FLAT PLATE
2	1740.3574	LBS/INCH	1836.0770	LBS	FLAT PLATE
3	2670.5484	LBS/INCH	5474.6242	LBS	FLAT PLATE
4	1740.3574	LBS/INCH	1836.0770	LBS	FLAT PLATE
5	2670.5484	LBS/INCH	5474.6242	LBS	FLAT PLATE
6	1740.3574	LBS/INCH	1836.0770	LBS	FLAT PLATE
7	2670.5484	LBS/INCH	5474.6242	LBS	FLAT PLATE

AXIAL STRAIN IS 3.15854E-03

FINAL RESULTS (LBS)

CRITICAL LOAD = 42028.130

MODE M= 6

GENERATE BUCKLING DETERMINANT FOR EIGENVECTOR SOLUTION
FOR A CRITICAL LOAD 2670.548
AND A CRITICAL MODE 6

DEX IDX 5.26381E-01 780

TEST PANEL TYPE NO 5   INTEGRAL PANEL 5 STIFFENERS

TIMING

TOTAL EXECUTION TIME IS   51.428 CP-SECONDS

TIMING BREAKDOWN BY SUBROUTINES   (IN CP-SECONDS)

ROUTINE	TOTAL TIME	NO. OF CALLS	AVERAGE PER CALL
DB	50.186	135	.371748
DT	0.000	3358	0.000000
RGE	0.000	10378	0.000000
DBGENS	0.000	21060	0.000000
ZARK	1.606	282	.005695
DET	23.482	135	.173941
CDTM	0.000	178	0.000000

BUCKLING LOADS OF ORTHOTROPIC LAMINATED STIFFENED PLATES

LOADING --- UNAXIAL COMPRESSION
BOUNDARY CONDITIONS -- LOADED EDGES ARE SIMPLY SUPPORTED, UNLOADED SIDES
ARE FREE, SIMPLY SUPP., CLAMPED, OR SUPPORTED BY BEAM ELEMENT
BEAM ELEMENT HAS SIMPLY SUPP. ENDS AT ITS NEUTRAL AXIS

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TEST PANEL TYPE NO 5 INTEGRAL PANEL 5 STIFFENERS

****
* EIGENVECTOR *
* AND *
* RELATIVE DISPLACEMENTS *
****

BANDWIDTHS

START BLOCK ---
LOWER BANDWIDTH 13
UPPER BANDWIDTH 13

REPETITIVE BLOCK ---
LOWER BANDWIDTH 13
UPPER BANDWIDTH 13

END BLOCK ---
LOWER BANDWIDTH 13
UPPER BANDWIDTH 13

TOTAL BUCKLING DETERMINANT BANDWIDTH IS ---

LOWER BANDWIDTH 13
UPPER BANDWIDTH 13

TOTAL ARRAY SPACE REQ. FOR EIGENVECTOR 3696

START	BLOCK	---NUMBER OF ROWS	IDAY =	20
		---NUMBER OF COLUMNS	IDAX =	24
REPETITIVE	BLOCK	---NUMBER OF ROWS	IDMY =	16
		---NUMBER OF COLUMNS	IDMX =	24
END	BLOCK	---NUMBER OF ROWS	IDBY =	20
		---NUMBER OF COLUMNS	IDBX =	24
BANDWIDTH			IDTH =	27
TOTAL NUMBER OF ROWS			NORD =	88
TOTAL NUMBER OF DIAGONAL BLOCKS IN DET.			NBB =	5

CORE REQUIREMENTS

START BLOCK ARRAY SPACE 0000001510 OCT 840 DEC

REPETITIVE BLOCK ARRAY SPACE 0000001240 OCT 672 DEC

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END        BLOCK  ARRAY SPACE 0000001510 OCT        840 DEC

FOR MORE OR LESS    STIFFENERS ADJUST FIELD LENGTH ACCORDINGLY

MAX. NO. OF BLOCKS WITH 70K FIELDLENGTH RESTRICTION IS            24



*******************************************************


ASSUMING AN OVERLAY LENGTH OF       0000027000  OCT.

(EXCLUDING BLANK COMMON)

REQUIRED    FIELD LENGTH IS         0000036160  OCT.

FOR EIGENVECTOR SOLUTION


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DETERMINANT FROM DLKDET    ---DET = DEX*(2**IDX)

        DEX    IDX      5.26381E-01                780

DETERMINANT FROM DECOMPOSED MATRIX

        DEX    IDX      5.26379E-01                780
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EIGENVECTOR

ITERATION NUMBER 1
NORMALIZING FACTOR 1.6002842E+07
VECTOR

-6.8501343E-02	1.4626479E-02	4.9550348E-02	-1.1701264E-04
-1.1777752E-04	-7.3527369E-03	1.9384886E-05	3.1699739E-04
2.3736243E-01	1.3983271E-01	-6.1132075E-01	1.3663306E-01
-3.3448624E-05	1.8486690E-02	-1.5521548E-04	-1.9342617E-02
1.2758658E-01	-1.8444097E-02	-3.4410270E-02	-7.9799751E-03
1.8972112E-04	1.0079153E-02	1.6149950E-04	5.8062544E-03
-3.4848231E-01	-2.0529798E-01	8.9751909E-01	-2.0057619E-01
1.8506581E-05	-1.0128612E-02	8.8398017E-05	1.0470687E-02
-1.6046313E-01	1.8490529E-02	1.2314665E-02	1.4746984E-02
-2.2309042E-04	-1.0712161E-02	-2.1022647E-04	-9.1447904E-03
3.8827313E-01	2.2873936E-01	-1.0000000E+00	2.2347981E-01
1.0794071E-10	-1.8092717E-07	-2.5622249E-09	3.4353167E-07
1.6046316E-01	-1.4746994E-02	1.2314613E-02	-1.8490522E-02
2.1023180E-04	9.1450565E-03	2.2309585E-04	1.0712423E-02
-3.4848248E-01	-2.0529807E-01	8.9751952E-01	-2.0057628E-01
-1.8506281E-05	1.0128201E-02	-8.8402827E-05	-1.0469944E-02
-1.2758669E-01	7.9799941E-03	-3.4410225E-02	1.8444103E-02
-1.6149621E-04	-5.8060448E-03	-1.8971582E-04	-1.0078695E-02
2.3736276E-01	1.3983291E-01	-6.1132161E-01	1.3663326E-01
3.3448911E-05	-1.8487125E-02	1.5520981E-04	1.9343424E-02
6.8501450E-02	1.1701546E-04	4.9550367E-02	-1.4626493E-02
-1.9385921E-05	-3.1759842E-04	1.1778319E-04	7.3530030E-03

EIGENVECTOR

ITERATION NUMBER 2
NORMALIZING FACTOR 1.6107661E+08
VECTOR

-6.8501391E-02	1.4626483E-02	4.9550354E-02	-1.1701403E-04
-1.1777999E-04	-7.3527221E-03	1.9385100E-05	3.1729007E-04
2.3736250E-01	1.3983275E-01	-6.1132093E-01	1.3663309E-01
-3.3448799E-05	1.8486934E-02	-1.5521257E-04	-1.9343059E-02
1.2758667E-01	-1.8444115E-02	-3.4410307E-02	-7.9799765E-03
1.8971920E-04	1.0079201E-02	1.6149755E-04	5.8059929E-03
-3.4848266E-01	-2.0529818E-01	8.9752000E-01	-2.0057640E-01
1.8506449E-05	-1.0128422E-02	8.8400361E-05	1.0470339E-02
-1.6046320E-01	1.8490524E-02	1.2314609E-02	1.4747003E-02
-2.2309292E-04	-1.0712153E-02	-2.1022950E-04	-9.1450760E-03
3.8827313E-01	2.2873936E-01	-1.0000000E+00	2.2347980E-01
-8.1199655E-12	1.1598600E-08	1.4190592E-10	-2.1178052E-08
1.6046320E-01	-1.4747003E-02	1.2314608E-02	-1.8490524E-02
2.1022986E-04	9.1450994E-03	2.2309253E-04	1.0712127E-02
-3.4848267E-01	-2.0529819E-01	8.9752001E-01	-2.0057640E-01
-1.8506434E-05	1.0128402E-02	-8.8400617E-05	-1.0470301E-02
-1.2758667E-01	7.9799768E-03	-3.4410307E-02	1.8444115E-02
-1.6149783E-04	-5.8060095E-03	-1.8971886E-04	-1.0079178E-02
2.3736250E-01	1.3983276E-01	-6.1132095E-01	1.3663310E-01
3.3448791E-05	-1.8486921E-02	1.5521274E-04	1.9343035E-02
6.8501393E-02	1.1701402E-04	4.9550355E-02	-1.4626483E-02
-1.9385061E-05	-3.1729087E-04	1.1777977E-04	7.3527068E-03

EIGENVECTOR

ITERATION NUMBER 3

NORMALIZING FACTOR 1.5107666E+08

VECTOR

-6.8501391E-02	1.4626483E-02	4.9550354E-02	-1.1701403E-04
-1.1777999E-04	-7.3527221E-03	1.9385100E-05	3.1729007E-04
2.3736250E-01	1.3983275E-01	-6.1132093E-01	1.3663309E-01
-3.3448799E-05	1.8486934E-02	-1.5521257E-04	-1.9343059E-02
1.2758667E-01	-1.8444115E-02	-3.4410307E-02	-7.9799765E-03
1.8971920E-04	1.0079201E-02	1.6149755E-04	5.8059929E-03
-3.4848266E-01	-2.0529818E-01	8.9752000E-01	-2.0057640E-01
1.8506449E-05	-1.0128422E-02	8.8400361E-05	1.0470339E-02
-1.6046320E-01	1.8490524E-02	1.2314609E-02	1.4747003E-02
-2.2309292E-04	-1.0712153E-02	-2.1022950E-04	-9.1450760E-03
3.8827313E-01	2.2873936E-01	-1.0000000E+00	2.2347980E-01
-8.1199747E-12	1.1598614E-08	1.4190609E-10	-2.1178077E-08
1.6046320E-01	-1.4747003E-02	1.2314608E-02	-1.8490524E-02
2.1022986E-04	9.1450994E-03	2.2309253E-04	1.0712127E-02
-3.4848267E-01	-2.0529819E-01	8.9752001E-01	-2.0057640E-01
-1.8506434E-05	1.0128402E-02	-8.8400617E-05	-1.0470301E-02
-1.2758667E-01	7.9799768E-03	-3.4410307E-02	1.8444115E-02
-1.6149783E-04	-5.8060095E-03	-1.8971886E-04	-1.0079178E-02
2.3736250E-01	1.3983276E-01	-6.1132095E-01	1.3663310E-01
3.3448791E-05	-1.8486921E-02	1.5521274E-04	1.9343035E-02
6.8501393E-02	1.1701402E-04	4.9550355E-02	-1.4626483E-02
-1.9385061E-05	-3.1729087E-04	1.1777977E-04	7.3527068E-03

BUCKLING LOADS OF ORTHOTROPIC LAMINATED STIFFENED PLATES

LOADING --- UNAXIAL COMPRESSION
BOUNDARY CONDITIONS -- LOADED EDGES ARE SIMPLY SUPPORTED, UNLOADED SIDES
ARE FREE, SIMPLY SUPP., CLAMPED, OR SUPPORTED BY BEAM ELEMENT
BEAM ELEMENT HAS SIMPLY SUPP. ENDS AT ITS NEUTRAL AXIS

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TEST PANEL TYPE NO 5 INTEGRAL PANEL 5 STIFFENERS

RELATIVE DISPLACEMENTS

P-VALUES ---ROOTS OF EQUILIBRION EQUATIONS---

ELEMENT TYPE NO. 1

PTT --REAL	--IMAGINARY	PIY
-1.30176513235563E+01	0.	5
8.50335671130288E+01	0.	6
3.59852257181947E+01	-1.03127591389610E+00	1
3.59852257181944E+01	1.03127591388856E+00	1

ELEMENT TYPE NO. 2

PTT --REAL	--IMAGINARY	PIY
-3.92158347529546E+01	0.	5
1.11231750542427E+02	0.	6
3.59852257181954E+01	-1.03127591391060E+00	1
3.59852257181935E+01	1.03127591390788E+00	1

RELATIVE DISPLACEMENTS

START SECTION --BLOCK NO. 1

ELEMENT NO. 1 TYPE PLATE EL. WIDTH 2.0500

Y-COORD.	W-DISPL.	V-DISPL.
-1.0250	-.00000	-.00042
-.5125	-.07294	-.00049
.0000	-.12249	-.00073
.5125	-.11822	-.00114
1.0250	.00293	-.00166

ELEMENT NO. 2 TYPE PLATE EL. WIDTH 1.0550

Y-COORD.	W-DISPL.	V-DISPL.
-.5275	.00166	.00293
-.2637	.27924	.00269
.0000	.75119	.00249
.2638	1.28542	.00235
.5275	1.84106	.00221

REPETITIVE SECTION --BLOCK NO. 2

ELEMENT NO. 3 TYPE PLATE EL. WIDTH 2.0500

Y-COORD.	W-DISPL.	V-DISPL.
-1.0250	.00293	-.00166
-.5125	.16373	-.00063
.0000	.22875	.00031
.5125	.19449	.00129
1.0250	-.00161	.00233

ELEMENT NO. 4 TYPE PLATE EL. WIDTH 1.0550

Y-COORD.	W-DISPL.	V-DISPL.
-.5275	-.00233	-.00161
-.2637	-.40992	-.00147
.0000	-1.10284	-.00137
.2638	-1.88719	-.00129
.5275	-2.70297	-.00121

REPETITIVE SECTION --BLOCK NO. 3

ELEMENT NO. 3 TYPE PLATE EL. WIDTH 2.0500

Y-COORD.	W-DISPL.	V-DISPL.
-1.0250	-.00161	.00233
-.5125	-.21975	.00106
.0000	-.28769	-.00012
.5125	-.23076	-.00131
1.0250	.00000	-.00261

ELEMENT NO. 4 TYPE PLATE EL. WIDTH 1.0550

Y-COORD.	W-DISPL.	V-DISPL.
-.5275	.00261	.00000
-.2637	.45673	.00000
.0000	1.22677	.00000

.2638	2.10268	.00000
.5275	3.01160	.00000

REPETITIVE SECTION --BLOCK NO. 4

ELEMENT NO. 3 TYPE PLATE EL. WIDTH 2.0500

Y-COORD.	W-DISPL.	V-DISPL.
-1.0250	-.00000	-.00261
-.5125	.23076	-.00131
.0000	.28769	-.00012
.5125	.21975	.00106
1.0250	.00161	.00233

ELEMENT NO. 4 TYPE PLATE EL. WIDTH 1.0550

Y-COORD.	W-DISPL.	V-DISPL.
-.5275	-.00233	.00161
-.2637	-.40992	.00147
.0000	-1.10284	.00137
.2638	-1.88719	.00129
.5275	-2.70297	.00121

END SECTION --BLOCK NO. 5

ELEMENT NO. 5 TYPE PLATE EL. WIDTH 2.0500

Y-COORD.	W-DISPL.	V-DISPL.
-1.0250	.00161	.00233
-.5125	-.19449	.00129
0.0000	-.22875	.00031
.5125	-.16373	-.00063
1.0250	-.00293	-.00166

ELEMENT NO. 6 TYPE PLATE EL. WIDTH 1.0550

Y-COORD.	W-DISPL.	V-DISPL.
-.5275	.00166	-.00293
-.2637	.27924	-.00269
.0000	.75119	-.00249
.2638	1.28542	-.00235
.5275	1.84106	-.00221

ELEMENT NO. 7 TYPE PLATE EL. WIDTH 2.0500

Y-COORD.	W-DISPL.	V-DISPL.
-1.0250	-.00293	-.00166
-.5125	.11822	-.00114
0.0000	.12249	-.00073
.5125	.07294	-.00049
1.0250	-.00000	-.00042

TIME FOR EIGENVECTOR SOLUTION IS 1.268 CP-SEC.

APPENDIX A

Generalized Eigenproblem for Large Matrices with a Repeated Block Structure

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A.1 Problem Definition

The analysis of buckling loads for stiffened panels gives rise to large, sparse, square matrices (G) which have a repeated block structure as illustrated in Figure A-1. The matrix G is a function of the load (λ) and the problem is to calculate the critical load; i.e., the smallest positive value of λ for which the equation

$$G(\lambda)x = 0$$

has a nontrivial solution. The eigenvector (x) corresponding to the critical load is also desired in some cases. This problem constitutes the general eigenproblem.

The ensuing sections of this appendix describe the numerical methods used for computing the critical load and the corresponding eigenvector.

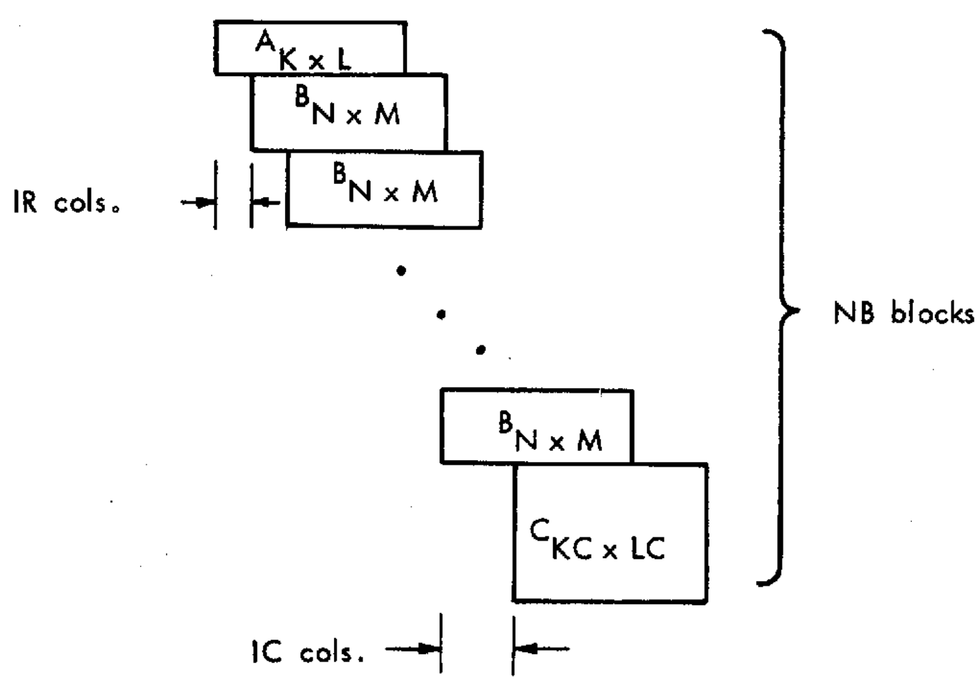

FIGURE A-1. Block Structure of Load Matrix for Stiffened Panels

A.2 Critical Load Computation

In accordance with the theory of linear equations, a value of λ for which the equation

$$G(\lambda)x = 0$$

has a nontrivial solution is equivalent to finding λ such that

$$\det G(\lambda) = 0 .$$

Thus, the critical load problem reduces to that of finding the smallest positive root for the real valued determinant function. This requires the use of an iterative scheme where determinant values corresponding to systematically refined root estimates are repeatedly evaluated.

While there are numerous well known techniques (Ref. 1) for finding roots, there are two particular difficulties with the structural buckling load problems:

(a) The matrices G are typically very large so that it is necessary to take advantage of their special structure in order to minimize the computation time and storage.

(b) Determinant values can easily exceed the floating-point range of the computer.

Subroutine BLKDET (see Ref. 2, Section 3.6) was designed to take advantage of the special structure of the G matrices. This subroutine can accommodate arbitrarily large matrices entirely within the computer core storage, provided only that the core can contain the submatrices A, B, and C (Figure A-1) and a modest amount of working space.

The problem of limited range for floating point numbers was handled in BLKDET by representing the determinant value with two numbers a and b where

$$\det G(\lambda) = a \cdot 2^b$$

and where

$$0.0625 \leqslant |a| < 1 .$$

The smallest positive root λ is initially determined within a broad interval by a stepping procedure. A sequence of steps are taken away from an input initial estimate of λ . The determinant values are tested for sign reversals, so that, ultimately an interval containing a root is established. Whether the root thus isolated is the smallest one or not depends on the initial estimate of λ .

Subroutine DETZER, (see Ref. 2, Section 3.9), which is designed to accept the two numbers a and b from BLKDET, further isolates the root within the interval established by the foregoing stepping procedure. The DETZER procedure is particularly efficient in obtaining the high precision roots required to satisfy eigenvector convergence criteria (see Section A.3 below).

The entire procedure for finding the critical load is summarized in Figure A-2.

An exact root satisfies the equation

$$\det G(\lambda) = 0 ;$$

however, in the actual computation exact roots are seldom obtained. A root λ is determined such that

$$\lambda_a < \lambda < \lambda_b$$

and

$$\det G(\lambda_a) \cdot \det G(\lambda_b) \leq 0$$

where

$$(\lambda_b - \lambda_a) < \epsilon$$

and ϵ is an acceptable error.

When λ satisfies the foregoing conditions, then it is correct to within $\pm\epsilon$ no matter how large $\det G(\lambda)$ may be.

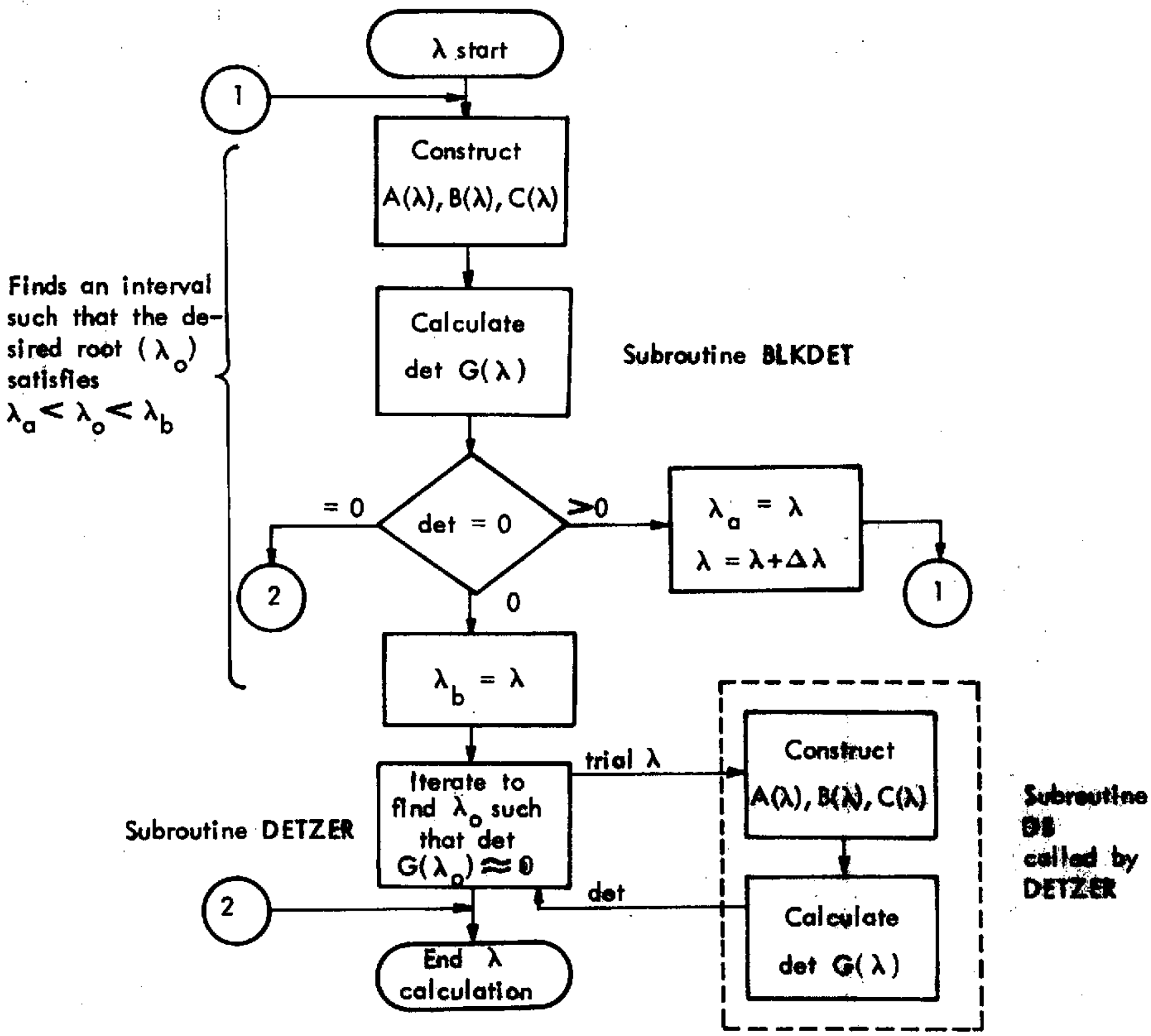

FIGURE A-2. Eigenvalue Computation

A.3 Eigenvector Computation

The method used to compute the eigenvector corresponding to a given root λ is Wielandt's method of inverse iteration (Ref. 3).

A stable method is of prime importance. Wilkinson [3] shows Wielandt's method to be stable for the standard eigenproblem

$$(A - \mu I)Z = 0 \ .$$

Stability for the general eigenproblem can be established from the stability of the standard eigenproblem as follows.

The standard eigenvalues and eigenvectors of a perturbed matrix $A + \epsilon B$ converge to those of A as $\epsilon \rightarrow 0$ (Wilkinson [3, pp. 66-67]); i.e.,

$$(A + \epsilon B - \mu I)Z \rightarrow (A - \mu I)Z = 0 \quad \text{as} \quad \epsilon \rightarrow 0.$$

If λ_k is an eigenvalue of the general eigenproblem $G(\lambda)x = 0$, then the eigenvector Z corresponding to $\mu = 0$ in the standard eigenproblem

$$[G(\lambda_k) - \mu I]Z = 0$$

is precisely the eigenvector x_k satisfying

$$G(\lambda_k)x = 0 \ .$$

Thus, assuming that λ_k^* is an approximation to λ_k, then $G(\lambda_k^*)$ satisfies

$$G(\lambda_k^*) = G(\lambda_k) + \epsilon B$$

for some scalar ϵ and matrix B, and

$$[G(\lambda_k^*) - \mu I]Z \rightarrow [G(\lambda_k) - \mu I]Z = 0$$

as $\lambda_k^* \rightarrow \lambda_k$.

Thus it can be concluded that the general eigenvector problem is equivalent to the standard eigenvector problem where the vector of interest corresponds to a zero eigenvalue, and that stability of Wielandt iteration for the standard eigenproblem implies stability for the general eigenproblem.

The implementation of the Wielandt method in the subroutine EIGV is described in Ref. 2, Section 3.13. A brief summary of Wielandt iteration for the standard eigenproblem is given below:

The solution of

$$AX = \mu Z$$

consists of repeated solutions of

$$(A - \mu^* I)v^{(i)} = b^{(i)}$$

where μ^* is an approximate eigenvalue and $b^{(1)}$ is chosen in some manner (typically $b^{(1)} = [1, 1, \ldots, 1]^T$). This is equivalent to repeatedly solving

$$G(\lambda^*)x^{(i)} = b^{(i)}$$

for the general eigenproblem.

The justification of the Wielandt iteration method for the standard eigenproblem is to be found in Reference 3.

From a practical point of view, when the eigenvalues are determined to a sufficiently small ϵ as noted in A.2, then eigenvector convergence is obtained in two iterations. A third iteration is frequently used to verify the convergence.

REFERENCES

1. Ralston, A., A First Course in Numerical Analysis, McGraw Hill, 1965.

2. Oeverli, V., BUCLASP, A Computer Program for Uniaxial Compressive Buckling Loads of Orthotropic Laminated Stiffened Plates, Program Description Document, prepared for NASA Langley Research Center under Contract No. NAS1-8858, November, 1970.

3. Wilkinson, J. H., The Algebraic Eigenvalue Problem, Clarendon Press - Oxford, 1965.

APPENDIX B

Test Problems Specified by NASA

1. a. Orthotropic Flange-Web

Buckling of a simply supported web with an orthotropic flange should be studied. Two configurations should be investigated, one which buckles locally, and the other which buckles in a beam-column mode. Results for local buckling should be compared with Figure 6 of Reference 1. Orthotropic properties assigned to the boron layer should be as follows:

$$E_{11} = 30.25 \times 10^6 \quad \text{(filament direction)}$$

$$E_{22} = 2.03 \times 10^6$$

$$\nu_{12} = .346 \qquad \text{(units - p.s.i.)}$$

$$G_{12} = .5249 \times 10^6$$

b. Results for the beam-column mode should be compared with the following result:

For Figure 7 (ref.), if the web is simply supported and $\frac{t_F}{t_w} = 3$, $\frac{b_w}{t_w} = 30$, the buckling coefficient k_W for a section with length $\frac{L}{b_w} = 10$ is approximately 1.56.

2. Truss-Core Sandwich Plate

Local buckling of a plate with equal-width elements should be investigated and compared with results presented in Reference 2. In addition, two truss-core sandwiches designed to buckle locally in one case with the core restraining the face, and in the other case with the face restraining the core. Results should be compared with these presented in Figure 5(a) of Reference 2.

3. <u>Discretely Stiffened Plate</u>

Buckling of a simply supported plate with a single eccentric beam stiffener should be investigated. Two configurations should be studied one with a deep stiffener (asymmetric buckling) and one with a shallow stiffener (symmetric buckling). Results should be compared with those presented in Figure 5 of Reference 1 and with Reference 3.

4. <u>Plate with Multiple Stiffeners</u>

T-section and integral stiffened plates with at least 5 stiffeners should be studied. The T-section plate should be sized to buckle locally and results compared with Reference 4. The integrally stiffened plate should be sized to buckle by general instability and results compared with Equation (A3) of Reference 5.

In all of the cases described in Paragraphs 1 to 4, it would be desirable (where possible) to investigate the buckling mode shape as well.

REFERENCES

1. Peterson, J. P.: Structural Efficiency of Aluminum Multiweb Beams and Z-stiffened Panels Reinforced with Filamentary Boron-Epoxy Composite, NASA TN D-5856, 1970.

2. Anderson, M. S.: Local Instability of the Elements of a Tross-Core Sandwich Plate, NASA TR R-30, 1959.

3. Seide, B.: The Effect of Longitudinal Stiffeners Located on One Side of a Plate on the Compressive Buckling Stress of the Plate-Stiffener Combination, NACA TN-2873, 1953.

4. Becker, H.: Handbook of Structural Stability. Part II. Buckling of Composite Elements, NACA TN-3782, 1957. (Figure 14d or 14e)

5. Block, D.; Card, M.; and Mikulas, M., Jr.: Buckling of Eccentrically Stiffened Orthotropic Cylinders, NASA TN D-2960, 1965.

APPENDIX C

CONVERSION OF UNITS USED IN THIS DOCUMENT TO SI UNITS

Conversion factors(1) for the units used in this document are given in the following table:

TABLE C.1

CONVERSION FACTORS

Physical quantity	Units used in this document	Conversion factor *	SI Unit
Length	in.	0.0254	meters (m)
Stress, modulus	lbs/in.^{2}	6.895×10^3	newtons/meter^2 (N/m^2)
Load per unit length (pounds per linear inch, PLI)	lbs/in.	1.751×10^5	newtons/meter (N/m)

*Multiply units used in this document by conversion factor to obtain equivalent value in SI units.

(1) Comm. on Metric Pract.: ASTM Metric Practice Guide NBS Handbook 102, U. S. Department Commerce, March 10, 1967.